PROBLEM 4.1:

A-PD

(a) $x[n] = x(nT_s) = 10\cos(880\pi n T_s + \varphi)$ $\qquad T_s = 0.0001$

$$880T_s = 880 \times 10^{-4} = 0.088 = 11/125$$

To find the number of samples within one period of the continuous cosine $x(t)$, find the largest integer satisfying $\quad 880\pi n T_s \leq 2\pi$

$$n \leq \frac{2}{0.088} = \frac{250}{11} = 22.73$$

There are 23 samples in one period, because samples $n = 0, 1, 2, \ldots 22$ are within one period.

NOTE: the period of $x[n]$ is <u>not</u> 23; it is actually 250.

(b) $y[n] = 10\cos(\omega_0 n T_s + \varphi)$

To get the same samples for $x[n]$ & $y[n]$ we solve:

$$\omega_0 n T_s = 880\pi n T_s + 2\pi \ell n \qquad \ell = \text{integer}$$

$$\Rightarrow \quad \omega_0 = 880\pi + \frac{2\pi\ell}{T_s} \qquad \frac{2\pi}{T_s} = 20{,}000\pi$$

Take $\ell = 1$: $\quad \omega_0 = 20{,}880\pi$

(c) Find largest integer satisfying

$$(20{,}880\pi)\, n T_s \leq 2\pi$$

$$n \leq \frac{2}{2.088} \quad \text{which is less than one!}$$

$\therefore$ only one sample per period is taken

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PROBLEM 4.2:

$$x(t) = 7\sin(11\pi t) = 7\cos(11\pi t - \pi/2) \longrightarrow \boxed{A/D} \longrightarrow x[n] = A\cos(\hat{\omega}_0 n + \varphi).$$

$$f_s.$$

(a) $f_s = 10$ samples/sec.

$$x(t)\Big|_{t=n/f_s} = x\left(\frac{n}{f_s}\right) = 7\cos\left(\frac{11\pi n}{10} - \pi/2\right).$$

$$= 7\cos\left(\frac{11\pi n}{10} - 2\pi n - \pi/2\right).$$

$$= 7\cos\left(-\frac{9\pi n}{10} - \pi/2\right) = 7\cos(.9\pi n + \pi/2).$$

$$\boxed{A = 7,\ \hat{\omega}_0 = 0.9\pi,\ \varphi = \pi/2}$$

(b) $f_s = 5$ samples/sec

$$x(t)\Big|_{t=n/f_s} = x\left(\frac{n}{5}\right) = 7\cos\left(\frac{11\pi n}{5} - \pi/2\right)$$

$$= 7\cos\left(\frac{\pi n}{5} - \frac{\pi}{2}\right)$$

$$\boxed{A = 7,\ \hat{\omega}_0 = \frac{\pi}{5},\ \varphi = -\frac{\pi}{2}}$$

(c) $f_s = 15$ samples/sec

$$x(t)\Big|_{t=n/f_s} = x\left(\frac{n}{15}\right) = 7\cos\left(\frac{11\pi n}{15} - \frac{\pi}{2}\right)$$

$$A = 7,\ \hat{\omega}_0 = \frac{11\pi}{15} = 2\pi\left(\frac{5.5}{15}\right)\ \vdots\ \varphi = -\pi/2$$

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PROBLEM 4.3:

$$x[n] = 2.2\cos(0.3\pi n - \pi/3) \qquad \boxed{f_s = 6000}$$

Compare to

$$x\left(\frac{n}{f_s}\right) = A\cos\left(2\pi f_0 \frac{n}{f_s} + \varphi\right)$$

sampled continuous-time signal.

$$\Rightarrow \frac{2\pi f_0}{f_s} = 0.3\pi, \text{ or } 0.3\pi + 2\pi, \text{ or } 0.3\pi - 2\pi.$$

Solve: $\frac{2\pi f_0}{f_s} = 0.3\pi \Rightarrow f_0 = f_s\left(\frac{0.3}{2}\right) = 6000 \times 0.15$

$$f_0 = 900 \text{ Hz}$$

NOTE: difference is f_s

$$\rightarrow x(t) = 2.2\cos(1800\pi t - \pi/3)$$

Then $\frac{2\pi f_0}{f_s} = 2.3\pi \Rightarrow f_0 = f_s\left(\frac{2.3}{2}\right) = 6900 \text{ Hz}$

$$\rightarrow x(t) = 2.2\cos(2\pi(6900)t - \pi/3).$$

Finally,

$$\frac{2\pi f_0}{f_s} = -1.7\pi \Rightarrow f_0 = f_s\left(-\frac{1.7}{2}\right) = -5100 \text{ Hz}.$$

$$x(t) = 2.2\cos(2\pi(-5100)t - \pi/3)$$

$$\rightarrow x(t) = 2.2\cos(2\pi(5100)t + \pi/3)$$

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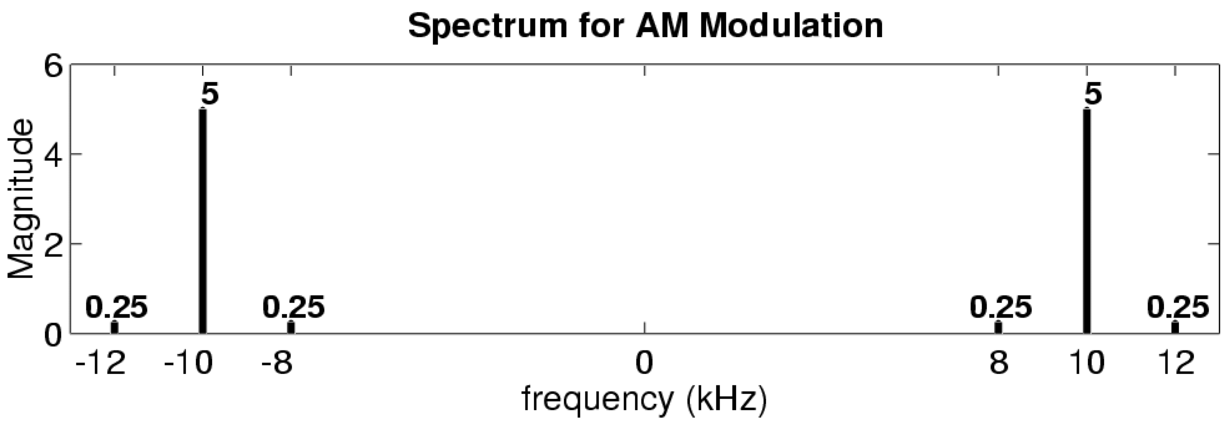
PROBLEM 4.4:

(a) $x(t) = \left[10 + \frac{1}{2}e^{j2\pi(2000)t} + \frac{1}{2}e^{-j2\pi(2000)t}\right]\left(\frac{1}{2}e^{j2\pi\times 10^4 t} + \frac{1}{2}e^{-j2\pi\times 10^4 t}\right)$

There are six terms:

$$x(t) = 5e^{j2\pi\times 10^4 t} + 5e^{-j2\pi\times 10^4 t} + \frac{1}{4}e^{j2\pi(12000)t} + \frac{1}{4}e^{-j2\pi(12000)t} + \frac{1}{4}e^{j2\pi(8000)t} + \frac{1}{4}e^{-j2\pi(8000)t}$$

Spectrum plot was created in MATLAB:



(b) Yes the waveform is periodic. The six frequencies $\{-12{,}000, -10{,}000, -8000, 8000, 10000, 12000\}$ are all divisible by 2000 Hz. Therefore, $f_o = 2000$ Hz is the fundamental frequency. The period is $1/f_o = 1/2000$ sec $= \frac{1}{2}$ msec

(c) The sampling rate must be greater than twice the highest frequency in $x(t)$.

$$\Rightarrow \quad f_s > 2(12{,}000) = 24{,}000 \text{ Hz}$$

PROBLEM 4.5:

(a) Let $x(t) = 10\cos(\omega_o t + \varphi)$

Sampling at a rate of $f_s \Rightarrow x[n] = x(t)\big|_{t=n/f_s} = x(n/f_s)$

$x[n] = 10\cos\left(\omega_o \frac{n}{f_s} + \varphi\right)$ $\qquad \boxed{f_s = 1000}$

Equate this to

$x[n] = 10\cos(0.2\pi n - \pi/7)$

$\frac{\omega_o}{f_s} = 0.2\pi \Rightarrow \omega_o = 0.2\pi \times 1000 = 200\pi$

$\varphi = -\pi/7$

A second possible signal is the "folded alias" at $(f_s - f_o)$

$$f_s - f_o = f_s - \frac{\omega_o}{2\pi} = 1000 - \frac{200\pi}{2\pi} = 900\text{ Hz}$$

In this case, the phase (φ) changes.

$$\tilde{x}(t) = 10\cos(2\pi(f_s - f_o)t + \psi)$$

$$\tilde{x}[n] = 10\cos\left(2\pi(f_s - f_o)\frac{n}{f_s} + \psi\right) = 10\cos\left(2\pi n - 2\pi f_o \frac{n}{f_s} + \psi\right)$$

$$= 10\cos\left(-\frac{2\pi f_o n}{f_s} + \psi\right) = 10\cos\left(2\pi \frac{f_o}{f_s} n - \psi\right).$$

$\boxed{f_o \text{ is still } 100\text{ Hz}}$

$\Rightarrow \psi = +\pi/7$

(b) Reconstruction of $x[n]$ with $f_s = 2000$ samples/sec.

The discrete and continuous domains are related by: $\frac{n}{f_s} \leftrightarrow t$ or $n \leftrightarrow f_s t$

So we replace "n" in $x[n]$ with $f_s t$. This is what an <u>ideal</u> D-to-A would do.

$$x[n] = 10\cos(0.2\pi n - \pi/7)$$

$$x(t) = 10\cos(0.2\pi f_s t - \pi/7) \quad \leftarrow f_s = 2000$$

$$= 10\cos(400\pi t - \pi/7)$$

$\omega_o = 400\pi \Rightarrow f_o = 200\text{ Hz}.$

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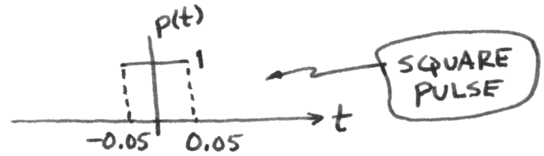
PROBLEM 4.6:

(a) $p(t) = \begin{cases} 1 & -0.05 \le t \le 0.05 \\ 0 & \text{otherwise} \end{cases}$

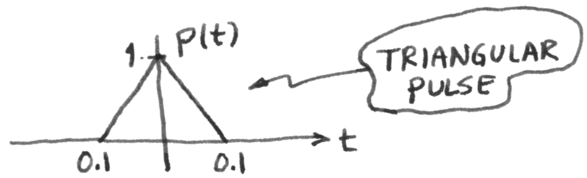


In the formula for $y(t)$

$$y(t) = \dots + y[0]\,p(t) + y[1]\,p(t-T_s) + y[2]\,p(t-2T_s) + \dots$$

The square pulses will <u>not</u> overlap, so the values of $y[n]$ will be extended over an interval of T_s.

(b) $p(t) = \begin{cases} 1 - 10|t| & -0.1 \le t \le 0.1 \\ 0 & \text{otherwise} \end{cases}$

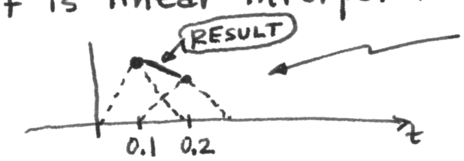


In this case, the neighboring terms do overlap

$$y(t) = \dots + y[0]\,p(t) + y[1]\,p(t-T_s) + y[2]\,p(t-2T_s) + \dots$$

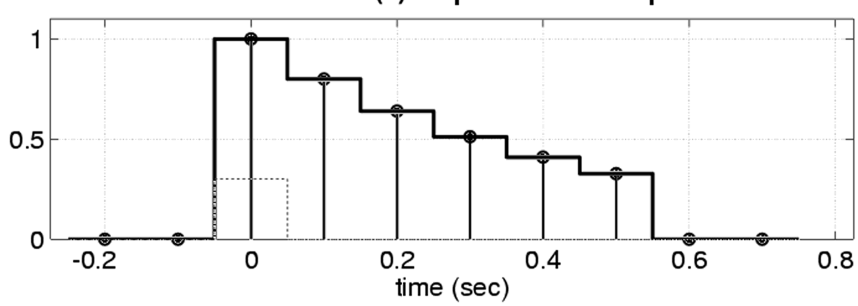
The result is linear interpolation.

Example:

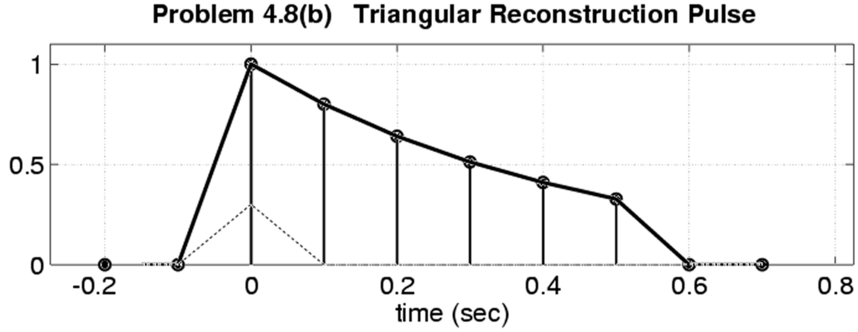


When we add these two triangles, the result between $t=0.1$ and $t=0.2$ is a straight line.

Problem 4.8(a) Square Pulse Shape



Problem 4.8(b) Triangular Reconstruction Pulse



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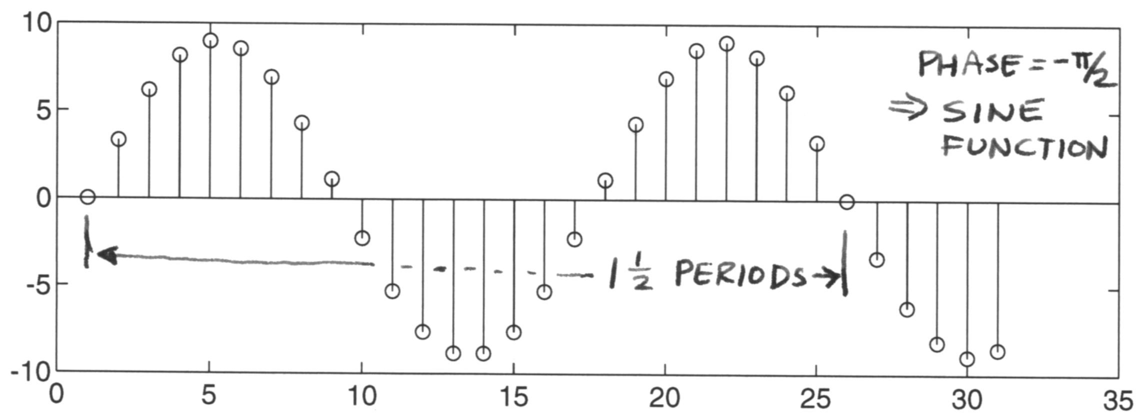
PROBLEM 4.7:

(a)



NOTE $1\frac{1}{2}$ PERIODS = 25 samples.

$\Rightarrow$ PERIOD = $50/3$ $\Rightarrow$ $\hat{\omega}_0 = 2\pi(0.06)$

$1/(50/3) = \frac{3}{50}$

(b) The vector xx is actually $x[n]$

$$x[n] = 9\cos(2\pi(394)n(0.01) + \pi/2)$$
$$= 9\cos(2\pi(3.94)n + \pi/2)$$
$$= 9\cos(2\pi(0.94)n + \pi/2)$$

REMOVED MULTIPLE OF 2π

$$= 9\cos(2\pi(-0.06)n + \pi/2)$$
$$x[n] = 9\cos(2\pi(0.06)n - \pi/2)$$

(c) Derivation in part (b) shows that $x[n]$ looks like samples of a 6 Hz sinusoid taken at $f_s = \frac{1}{0.01} = 100$ Hz.

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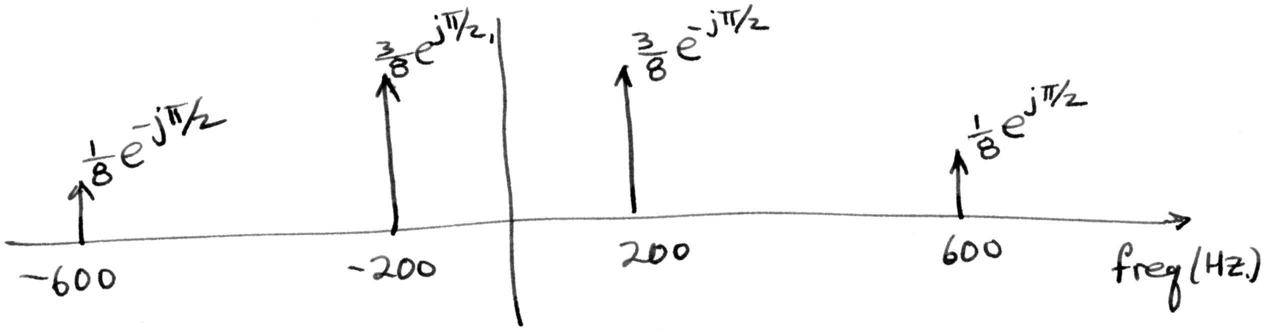
PROBLEM 4.9:

(a) Draw a sketch of the spectrum of $x(t)$ which is "sine-cubed" $\quad x(t) = \sin^3(400\pi t)$

$$x(t) = \left(\frac{e^{j400\pi t} - e^{-j400\pi t}}{2j} \right)^3$$

$$= \frac{1}{-8j} \left\{ e^{j1200\pi t} - 3e^{j400\pi t} + 3e^{-j400\pi t} - e^{-j1200\pi t} \right).$$



$$\frac{1}{-8j} = \frac{1}{8} e^{j\pi/2}$$

(b) Determine the minimum sampling rate that can be used to sample $x(t)$ without any aliasing.

$$f_s \geq 2 f_{HIGH}$$

$$\Rightarrow f_s \geq 1200 \text{ Hz}$$

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PROBLEM 4.12:

(a) $x[n] = 10\cos(0.13\pi n + \pi/13)$ the sampling rate is $f_s = 1000$ samples/second

$.13\pi n = 2\pi(0.065)n = 2\pi(65)\frac{n}{1000}$ $\Rightarrow$ 65Hz is one Freq.

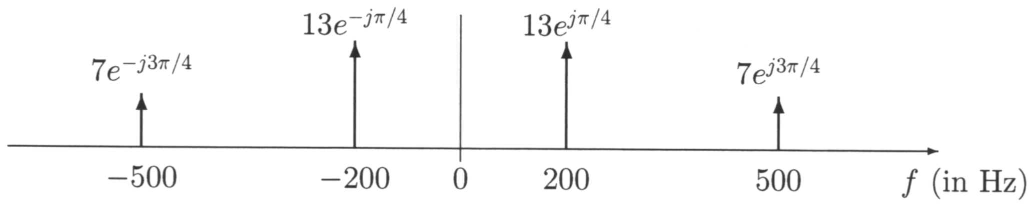
$x_1(t) = 10\cos(2\pi(65)t + \pi/13)$

Also, it could be "folded" case: $1000 - 65 = 935$ Hz

$x_2(t) = 10\cos(2\pi(935)t - \pi/13)$

NOTE phase reversal

(b) If the input $x(t)$ is given by the two-sided spectrum representation shown below, determine a $\boxed{\text{simple}}$ formula for $y(t)$ when $f_s = 700$ samples/sec. (for both the C/D and D/C converters).



$x(t) = 26\cos(2\pi(200)t + \pi/4) + 14\cos(2\pi(500)t + 3\pi/4)$

$x[n] = 26\cos(2\pi(\frac{2}{7})n + \pi/4) + 14\cos(2\pi(\frac{5}{7})n + 3\pi/4)$

THIS TERM FOLDS

$x[n] = 26\cos(2\pi(\frac{2}{7})n + \pi/4) + 14\cos(2\pi(\frac{-2}{7})n + 3\pi/4)$.

$\Rightarrow y(t) = 26\cos(2\pi(200)t + \pi/4) + 14\cos(2\pi(200)t - 3\pi/4)$.

$y(t) = 12\cos(2\pi(200)t + \pi/4)$

PROBLEM 4.13:

Assume that the sampling rates of a C-to-D and D-to-C conversion system are equal, and the input to the Ideal C-to-D converter is

$$x(t) = 2\cos(2\pi(50)t + \pi/2) + \cos(2\pi(150)t)$$

(a) If the output of the ideal D-to-C Converter is equal to the input $x(t)$, i.e.,

$$y(t) = 2\cos(2\pi(50)t + \pi/2) + \cos(2\pi(150)t)$$

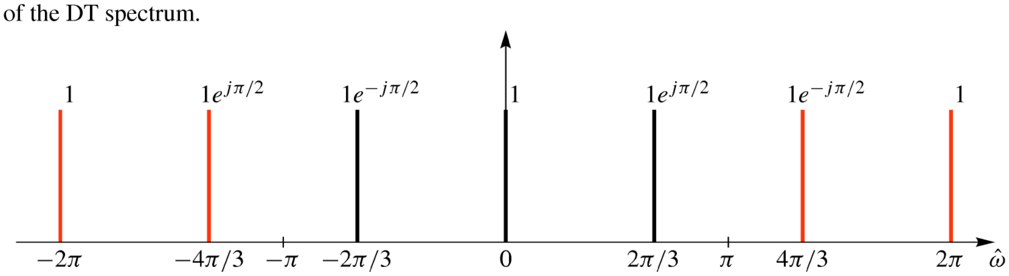
what general statement can you make about the sampling frequency f_s in this case?
Solution: The sampling frequency must be greater than twice the highest frequency, because there was no aliasing. Thus, we can say that

$$F_s > 2 \times 150 = 300 \text{ Hz}$$

(b) If the sampling rate is $f_s = 250$ samples/sec., determine the discrete-time signal $x[n]$, and give an expression for $x[n]$ as a sum of cosines. *Make sure that all frequencies in your answer are positive and less than π radians.* *Solution:* Replace t with $n/f_s = n/250$ to get

$$\begin{aligned} x[n] = x(n/250) &= 2\cos(2\pi(50)(n/250) + \pi/2) + \cos(2\pi(150)(n/250)) \\ &= 2\cos(2\pi(0.2)n + \pi/2) + \cos(2\pi(0.6)n) \\ &= 2\cos(2\pi(0.2)n + \pi/2) + \cos(2\pi(0.4)n) \end{aligned}$$

(c) Plot the spectrum of the signal in part (b) over the range of frequencies $-\pi \le \hat{\omega} \le \pi$. The plot below shows the periodicity of the DT spectrum.



(d) If the output of the Ideal D-to-C Converter is

$$y(t) = 2\cos(2\pi(50)t + \pi/2) + 1$$

determine the value of the sampling frequency f_s. (Remember that the input signal is $x(t)$ defined above.)
Solution: Since the frequency of 50 Hz is preserved, the other frequency of 150 Hz must have been aliased to 0 Hz. This can happen if the sampling frequency is $f_s = 150$ Hz, in which case the discrete-time signal is

$$\begin{aligned} x[n] = x(n/150) &= 2\cos(2\pi(50)(n/150) + \pi/2) + \cos(2\pi(150)(n/150)) \\ &= 2\cos(2\pi n/3 + \pi/2) + \cos(2\pi n) \\ &= 2\cos(2\pi n/3 + \pi/2) + 1 \end{aligned}$$

When $x[n]$ is reconstructed by the D/A converter running at $f_s = 150$ Hz, the final output will be

$$y(t) = x[n]\Big|_{n \to f_s t} = 2\cos(2\pi(150t)/3 + \pi/2) + 1 = 2\cos(2\pi(50)t + \pi/2) + 1$$

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