EEL 3135 Signals and Systems
Fall 2003
Professor Jian Li
100-MINUTE EXAMINATION # 1
October 7, 2003

There are 8 problems on the exam. Each problem counts 25 points. Do your work on the exam. This is an open-book and open-notes exam.

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Please read and sign the following statement:

I have neither given nor received aid on this examination.

Signed:
1. (25 points) This problem concerns the signal

\[ r(t) = A \cos(\omega_0 t + \phi) \]

with \( A = 10; \ \omega_0 = 2\pi(10); \ \phi = -\pi/2; \)

(a) What is the period \( T_0 \) of \( r(t) \)?

\[ T_0 = \frac{1}{\omega_0} \]

(b) In the space below, sketch \( r(t) \), making certain to label \( A, T_0 \) and \( \phi \).

(c) Is \( r(t) \) an even function, an odd function, or neither an even nor an odd function of \( t \)?

\( \text{odd} \)

(d) Find \( \tau \) if \( r(t) \) is written in the form

\[ r(t) = A \cos(\omega_0(t - \tau)) \]

\[-\frac{\pi}{2} - \phi = -\omega_0 \tau = -2\pi(10) \tau \]

\[ \tau = \frac{1}{40} \]
2. (25 points) This problem concerns the signal

\[ x(t) = A_1 \cos(\omega_0 t + \phi_1) + A_2 \cos(\omega_0 t + \phi_2) \]

with \( A_1 = 8; \quad A_2 = 10; \quad \omega_0 = 2\pi(10); \quad \phi_1 = \pi/4; \quad \phi_2 = -3\pi/4 \)

(a) Determine the phasors of the two sinusoids.

\[ X_1 = 8 \ e^{j \frac{\pi}{4}} \]
\[ X_2 = 10 \ e^{-j \frac{3\pi}{4}} \]

(b) Determine \( A, \omega, \) and \( \phi \) if \( x(t) \) is written as

\[ x(t) = A \cos(\omega t + \phi) \]

\[ X_1 + X_2 = 8 \ e^{j \frac{\pi}{4}} + 10 \ e^{-j \frac{3\pi}{4}} = 2 \ e^{j \frac{\pi}{4}} \]

\( A = 2, \quad \omega = 2\pi(10), \quad \phi = -\frac{3\pi}{4} \)
3. (25 points) A signal is given by

\[ y(t) = 10 \cos(35\pi t + \pi/4) \]

(a) What is the minimum sampling frequency for \( y(t) \) for perfect reconstruction?

\[ f_s > \frac{2}{11} \text{ samples/second} \]

(b) If \( y(t) \) is sampled with a sampling frequency of 10 Hz, express the sampled signal \( y[n] \) in the form \( y[n] = A \cos(\omega_0 n + \phi) \) by finding \( A, \omega_0, \) and \( \phi \).

\[ \omega_0 = \frac{2\pi}{10} \left(\frac{35}{i} \right) + 2\pi k = 3.5\pi + 2\pi l \]

\[ l = -2, \quad \omega_0 = -0.5\pi \in [-\pi, \pi \rangle. \]

\[ y[n] = 10 \cos \left( -0.5\pi n + \frac{\pi}{4} \right) \]

\[ = 10 \cos \left( 0.5\pi n - \frac{\pi}{4} \right). \]

(c) What is the smallest frequency of a sinusoid that will yield the same \( y[n] \)? Determine the sinusoid.

\[ y'[t + 1] = 10 \cos \left( 0.5\pi t + \frac{\pi}{4} \right) \]

\[ = 10 \cos \left( 5\pi t - \frac{\pi}{4} \right). \]

Smallest frequency is \( \frac{5}{2} \text{ Hz}. \)
(25 points) Consider the signal

\[ q(t) = 4 \cos(2\pi3000t) + 4 \cos(2\pi1000t - \pi/4) + 4 \sin(2\pi4000t - \pi/4) \]

(a) Is \( q(t) \) periodic? If so, what is the fundamental period? If not, why not?

Yes. \[ q_{ed}(3000, 1000, 4000) = (000 - f_0) \]

\[ T_0 = \frac{1}{1000}. \]

(b) Determine the Fourier series coefficients \( a_k \), \( k = 0, \pm 1, \pm 2, \ldots \).

\[ f_k(t) = 4 \cos(2\pi1000t - \frac{\pi}{4}) + 4 \cos(2\pi3000t) + 4 \cos(2\pi4000t - \frac{3\pi}{4}) \]

\[ a_1 = 2e^{j\frac{\pi}{4}}, \quad a_{-1} = 2e^{-j\frac{\pi}{4}} \]

\[ a_3 = 2, \quad a_{-3} = 2 \]

\[ a_4 = 2e^{j\frac{3\pi}{4}}, \quad a_{-4} = 2e^{-j\frac{3\pi}{4}} \]

All other \( a_k \) are zero.

(c) Plot the spectrum.

\[ 2e^{j\frac{\pi}{4}} \quad 2e^{j\frac{\pi}{4}} \quad 2 \quad 2e^{-j\frac{3\pi}{4}} \]

(d) What is the lowest sampling frequency which could be used to sample \( q(t) \) which will avoid aliasing?

\[ f_s > 2(4000) = 8000 \text{ samples/sec}. \]
5. (25 points) Consider the signal

\[ x[n] = 10 \cos(0.1\pi n - \pi/4) \]

(a) Determine two different continuous-time signals \( x_1(t) \) and \( x_2(t) \) whose samples are equal to \( x[n] \) when the sampling frequency is \( f_s = 100 \) samples/sec.

\[ \omega = \frac{2\pi f_s}{f_s} + 2\pi \lambda, \]

\[ 0.1\pi = \frac{2\pi f_s}{100} + 2\pi \lambda. \]

\[ \lambda = 0, \quad f_s = 5 \quad \text{Hz}, \quad \lambda = -1, \quad f_s = 105 \quad \text{Hz} \]

\[ x_1(t) = 10 \cos\left(2\pi \times 5 \times t - \frac{\pi}{4}\right) \]

\[ x_2 = 10 \cos\left(2\pi \times (105) \times t - \frac{\pi}{4}\right). \]

(b) Determine the signal \( y(t) \) reconstructed from \( x[n] \) by an ideal D-to-C converter operating at a sampling rate of 200 samples/sec.

\[ y(t) = 10 \cos\left(2\pi (200) \times t - \frac{\pi}{4}\right) \]

\[ = 10 \cos\left(2\pi \times t - \frac{\pi}{4}\right) \]

(c) Suppose that the input \( x(t) \) of the ideal C-to-D converter is

\[ x(t) = 10 \cos(2\pi(10)t - \pi/4) \]

Determine two different sampling frequencies so that the output of the C-to-D converter is \( x[n] \).

\[ 0.1\pi = \frac{2\pi f_s}{f_s} + 2\pi \lambda. \]

\[ \lambda = 0, \quad f_s = 200 \quad \text{samples/sec} \]

\[ \lambda = -1, \quad f_s = \frac{200}{2.1} \quad \text{samples/sec}. \]
6. (25 points) This problem concerns the signal $x(t)$ which is periodic with period 2.

- $x(t) = -2; \quad 0.1 < t < 0.2$
- $x(t) = 0; \quad 0 < t \leq 0.1 \text{ and } 0.2 < t \leq 2$
- $x(t) = x(t+2); \quad -\infty < t < \infty$

(a) Sketch $x(t)$ for $0 < t < 4$.

(b) Compute the Fourier Series for $x(t)$. You might find it useful to know that

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$a_k = \frac{1}{2} \int_{0}^{2} x(t) e^{-j \frac{2\pi}{2} k \frac{1}{2} t} dt$$

$$= \frac{1}{2} \int_{0}^{0.2} (-2) e^{-j \frac{2\pi}{2} k t} dt$$

$$= \frac{2}{2} \left[ \frac{e^{-j \frac{2\pi}{2} k t}}{-j \frac{2\pi}{2} k} \right]_{0.1}^{0.2}$$

$$= \frac{e^{-j \frac{2\pi}{2} k (0.2)}}{-j \frac{2\pi}{2} k} - \frac{e^{-j \frac{2\pi}{2} k (0.1)}}{-j \frac{2\pi}{2} k}$$

$$= -\frac{e^{-j \frac{\pi}{2} k (0.2)} - e^{-j \frac{\pi}{2} k (0.1)}}{-j \frac{\pi}{2} k}.$$
7. (25 points) This problem concerns the signal $x(t)$ which is periodic with period 10.

$$
\begin{align*}
    x(t) &= 2t; & 0 \leq t \leq 1 \\
    x(t) &= -2t; & -1 \leq t < 0 \\
    x(t) &= 0; & 1 < t \leq 5 \text{ and } -5 \leq t < -1 \\
    x(t) &= x(t + 10); & -\infty < t < \infty
\end{align*}
$$

(a) Sketch $x(t)$ for $-10 < t < 10$?

(b) Is $x(t)$ an even function, an odd function, or neither an even nor an odd function of $t$?

\[ \text{even} \]

(c) Compute the Fourier Series coefficient $a_0$ for $x(t)$.

$$
\begin{align*}
    a_0 &= \frac{1}{10} \int_{-5}^{5} x(t) \, dt \\
    &= \frac{1}{10} \left( 2 \times 1 \times \frac{1}{2} + 2 \times 1 \times \frac{1}{2} \right) \\
    &= \frac{1}{5}.
\end{align*}
$$
8. (25 points) A signal is given by
\[ y(t) = 2\delta(t + 1) + 2\delta(t - 1) \]

(a) Determine the Fourier transform of \( y(t) \).

\[ Y(j\omega) = 2 e^{-j\omega(-1)} + 2 e^{-j\omega(1)} \]
\[ = 4 \cos \omega \]

(b) Sketch the Fourier transform of \( y(t) \).