Problem 5 (5.3)

part (a)

\[
y[0] = 2x[0] - 3x[-1] + 2x[-2] = 2 \\
y[1] = 2x[1] - 3x[0] + 2x[-1] = 1 \\
y[3] = -1 \\
y[4] = 2 \\
y[5] = 3 \\
y[6] = 1 \\
y[7] = 1 \\
y[8] = 1 \\
y[9] = 1 \\
y[10] = 1
\]

<table>
<thead>
<tr>
<th>n</th>
<th>&lt;0</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>x[n]</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

| y[n] | 0 | 2 | 1 | 2 | -1 | 2 | 3 | 1 | 1 | 1 |

part (b) Please see plot on the next page (page 10)

part (c) \[ y[n] = h[n] = 2y[n] - 3y[n-1] + 2y[n-2] \]

\[
h[0] = 2(0) - 3(0) + 2(0) = 2 \\
h[1] = 2(0) - 3(1) + 2(0) = -3 \\
h[2] = 2(0) - 3(0) + 2(1) = 2 \\
\]

For all \( n > 2 \) \( h[n] = 0 \)

Please see the plot on the next page.

Note: \( h[n] \) just "read out" the filter coefficients

i.e. \( h[n] = b_n \).
Problem 5.2 (plots)

Plots via MATLAB

INPUT SIGNAL $x[n]$

OUTPUT SIGNAL $y[n]$

UNIT IMPULSE $\delta[n]$

IMPULSE RESPONSE $h[n]$
Problem 6 5-9

**Linearity**

(a) **YES**

Let \( x[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n] \)

\[ y[n] = (\alpha_1 x_1[n] + \alpha_2 x_2[n]) \cos (0.2 \pi n) \]

\[ = \alpha_1 x_1[n] \cos (0.2 \pi n) + \alpha_2 x_2[n] \cos (0.2 \pi n) \]

\[ y_1[n] + y_2[n] \]

\( x_2[n] \rightarrow y_2[n] \)

\( x_1[n] \rightarrow y_1[n] \)

(b) **YES**

\[ y[n] = (\alpha_1 x_1[n] + \alpha_2 x_2[n]) - (\alpha_1 x_1[n-1] + \alpha_2 x_2[n-1]) \]

\[ = \alpha_1 (x_1[n] - x_1[n-1]) + \alpha_2 (x_2[n] - x_2[n-1]) \]

\[ y_1[n] + y_2[n] \]

\( x_1[n] \rightarrow y_1[n] \)

\( x_2[n] \rightarrow y_2[n] \)

(c) **NO**

Let \( x_1[n] = \delta[n] \) and \( x_2[n] = -2 \delta[n] \)

\( y_1[n] = \delta[n] \)

\( y_2[n] = x_2[n] \)

\( |x_2[n]| = |\delta[n]| = 2 \delta[n] \)

Let \( x[n] = x_1[n] + x_2[n] = \delta[n] - 2 \delta[n] = -\delta[n] \)

\( y[n] = |x[n]| = \delta[n] \leftrightarrow y_1[n] + y_2[n] = \delta[n] + 2 \delta[n] = 3 \delta[n] \)

Since \( y[n] \neq y_1[n] + y_2[n] \rightarrow NO \)

(d) **NO** if \( B \neq 0 \)

If \( x_1[n] \rightarrow y_1[n] \), then \( 2 x_1[n] \rightarrow 2 y_1[n] \)

\[ A(2x_1[n]) + B = 2(Ax_1[n] + B) + B \neq 2 y_1[n] \]

**Time Invariant**

(a) **NO**!

Let \( x[n] = \delta[n] \), then \( y[n] = \delta[n] \cos (0.2 \pi n) = \delta[n] \) (when \( n = 0 \))

Now try

\( x[n-1] = \delta[n-1] \), then output \( \cos (0.2 \pi n) \delta[n-1] \)

But \( \cos (0.2 \pi n) \delta[n-1] \neq y[n-1] = \delta[n-1] \)
(b) YES

If \( x[n] \rightarrow y[n] \), let \( v[n] = x[n-n_0] \).

Output = \( v[n] - v[n-1] = x[n-n_0] - x[n-n_0-1] \).

This is the same as \( y[n-n_0] = x[n-n_0] - x[n-n_0-1] \).

(c) YES

Output depends only on \( v[n] \) at \( n \), so \( y[n-n_0] = x[n-n_0] \).

(d) YES

\( y[n-n_0] = A x[n-n_0] + B \) is always true.

**CAUSAL**

(a) YES

\( y[n] \) at \( n = n_0 \) depends only on \( x[n] \) at \( n = n_0 \), and not on past or future values.

(b) YES

\( y[n] \) at \( n = n_0 \) depends only on \( x[n] \) at \( n = n_0 + n-n_0-1 \), so it only uses "present" and the "past".

(c) YES

\( y[n] \) at \( n = n_0 \) depends only on \( x[n] \) at \( n = n_0 \).

\( y[n_0] = x[n_0] \).

(d) YES

\( y[n] \) at \( n = n_0 \) depends only on \( x[n] \) at \( n = n_0 \).

\( y[n_0] = A x[n_0] + B \).
Problems 7 5-10

part (a)

\[ x[n] = s[n] - s[n-1] \rightarrow y[n] = s[n] - s[n-1] + 2s[n-3] \]
\[ x[n] = \cos \left( \frac{\pi}{4} n / 2 \right) \rightarrow y[n] = 2 \cos \left( \frac{\pi}{4} n / 2 - \pi / 4 \right) \]

\[ y[n] \]

\[ -3 -2 -1 \]
\[ 1 2 \]
\[ 3 4 5 6 7 \]
\[ n \]

part (b)

\[ x[n] = 7s[n] - 7s[n-2] \]
\[ = 7[s[n] - s[n-2]] \]

In order to use linearity and time-invariance, we need to express \( x[n] \) in terms of known signals.

Let \( x_1[n] = s[n] - s[n-1] \)

Then \( x[n] = 7s[n] - 7s[n-2] = 7x_1[n] + 7x_1[n-1] \)

Now, LTI system \( \Rightarrow \)

\[ 7x_1[n] \rightarrow 7s[n] - 7s[n-1] + 14s[n-3] \]
\[ 7x_1[n-1] \rightarrow 7s[n-1] - 7s[n-2] + 14s[n-4] \]

Add them together

\[ x[n] \rightarrow 7s[n] - 7s[n-2] + 14s[n-3] + 14s[n-4] \]
Problem 8  5-14(a) and 5-15(c)

5-14(a)

$h[n] = \delta[n-2] + x[n] \rightarrow y[n] = u[n-3] - u[n-6]$

Here is how $y[n]$ looks:

$$y[n] = \begin{cases} 
1 & \text{if } 3 \leq n \leq 5 \\
0 & \text{otherwise}
\end{cases}$$

Since $h[n] = \delta[n-2]$, we know that if we feed $\delta[n]$ as the input, then the output is $\delta[n-2]$.

\[\begin{array}{c}
\delta[n] \rightarrow \delta[n-2] \\
\delta[n-1] \rightarrow \delta[n-3] \\
\delta[n-2] \rightarrow \delta[n-4] \\
\delta[n-3] \rightarrow \delta[n-5]
\end{array}\]

These are the ones that we need.

So,

$$x[n] = \delta[n-1] + \delta[n-2] + \delta[n-3]$$

5-15(c)

$$x[n] = \frac{1}{2}^n u[n]$$

$x[n] \rightarrow \delta[n-1]$

In order to find the FIR filter, we just need to find out how we have to manipulate...
\( x[n] \) to obtain \( s[n] \)

\[ x[n] = \frac{1}{2^n} u[n] \quad \text{which looks as follows} \]

\[ x[n] \]

\( n \)

\( \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots \)

In order to get \( s[n] \) we need to subtract the following sequence.

This signal is

\[ \frac{1}{2} \times x[n-1] \]

\( s[n] = x[n] - \frac{1}{2} x[n-1] \]

So \( h[n] = s[n-1] - \frac{1}{2} s[n-2] \), because

\[ \begin{align*}
  x[n] & \rightarrow s[n-1] + \\
  x[n-1] & \rightarrow s[n-2] + \\
  \frac{1}{2} x[n-1] & \rightarrow \frac{1}{2} s[n-2]
\end{align*} \]

we could do these because these are LTI systems.
part (a): The MATLAB program has two filters that are added together, and then filtered again.

\[ y_1[n] = x[n] + x[n-1] + x[n-2] + x[n-3] \]
\[ y_2[n] = x[n] - x[n-1] - x[n-2] + x[n-3] \]
\[ w[n] = y_1[n] + y_2[n] \]
\[ y[n] = w[n] + w[n-1] + w[n-2] \]

System 5,
\[ \{b_k\} = \{1,1,1\} \]

\[ S_1 : h_1[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] \]
\[ S_2 : h_2[n] = \delta[n] - \delta[n-1] - \delta[n-2] + \delta[n-3] \]
\[ S_3 : h_3[n] = \delta[n] + \delta[n-1] + \delta[n-2] \]

part (b) when \( x[n] = \delta[n] \), \( w[n] = h_1[n] + h_2[n] \)

\[ = 2\delta[n] + 2\delta[n-1] \]

Then \( y[n] = h_3[n] + w[n] \)

\[ = 2\delta[n] + 2\delta[n-1] + 2\delta[n-2] + 2\delta[n-3] + 2\delta[n-4] \]

\[ + 2\delta[n-5] \]

The overall difference equation is obtained by noting that the filter coefficients are equal to the impulse response values: \( b_k = h[k]|_{n=k} \)

\[ y[n] = 2x[n] + 2x[n-1] + 2x[n-2] + 2x[n-3] + 2x[n-4] \]
\[ + 2x[n-5] \]