NOTE: when finding \( \hat{\omega} \), make sure to choose a value of \( \theta \) such that \(-\pi < \hat{\omega} < \pi\)

Form A  
EEL 3135  
Section 1471  

Quiz #2

Given 
\[ x(t) = 6 \cos \left( 2\pi (50)t - \pi/3 \right) \]

(a) According to the Sampling Theorem, what should the sampling frequency be? (Give your answer in terms of the frequency of the given signal)?
\[ f_s > 2f_0 \]
\[ \therefore f_s > 2(50) \Rightarrow f_s > 100Hz \]

(b) Would aliasing occur if the signal \( (x(t)) \) is sampled at a frequency of 200Hz?  
What would the sampled signal \( (x[n]) \) be at that frequency?
\[ f_s' = 200Hz > 100Hz \]
\[ x[n] = 6 \cos (\hat{\omega} n - \pi/3) \]
\[ = 6 \cos \left( \frac{\pi}{2} n - \frac{\pi}{3} \right) \]
\[ \hat{\omega} = \frac{2\pi f_s}{f_s} + \frac{2\pi n}{f_s} \]
\[ = \frac{2\pi}{4} \Rightarrow \pi/2 \]

(c) If the \( x[n] \) in part (b) was given as input to an IDEAL D-to-C converter, what would the reconstructed signal \( x(t) \) be?
Since there was no aliasing, \( x(t) \) = the original signal
\[ \therefore x(t) = 6 \cos \left( 2\pi (50)t - \pi/3 \right) \]

(d) Would aliasing occur if the signal \( (x(t)) \) is sampled at a frequency of 60Hz?  
What would the sampled signal \( (x[n]) \) be at that frequency?
\[ f_s' = 60Hz < 100Hz \]
\[ \hat{\omega} = -\frac{2\pi n}{6} = -\pi/3 \]
\[ x[n] = 6 \cos \left( \hat{\omega} n - \frac{\pi}{3} \right) \]
\[ = 6 \cos \left( -\frac{\pi}{3} n - \frac{\pi}{3} \right) = 6 \cos \left( \frac{\pi}{3} n + \frac{\pi}{3} \right) \]

(e) If the \( x[n] \) in part (d) was given as input to an IDEAL D-to-C converter, what would the reconstructed signal \( x(t) \) be?
Discrete ↔ Continuous
\[ n \leftrightarrow \frac{tf_s}{3} \]
\[ x(t) = 6 \cos \left( \frac{\pi}{3} (60) t + \pi/3 \right) \]
\[ = 6 \cos \left( \frac{\pi}{3} (200) t + \pi/3 \right) = 6 \cos \left( 2\pi (10) t + \pi/3 \right) \]
NOTE: when finding \( \dot{\omega} \), make sure to choose a value of \( \dot{\omega} \) such that \(-\pi < \dot{\omega} < \pi\).

Form B
EEL 3135
Section 1471

Name: KEY
UF ID: ____________________________

Quiz #2

Given

\[ x(t) = 3 \cos(2\pi(20)t - \pi/4) \]

(a) According to the Sampling Theorem, what should the sampling frequency be? (Give your answer in terms of the frequency of the given signal)

\[ f_s \geq \frac{2\pi}{\dot{\omega}} \]

\[ f_s \geq \frac{40}{\sqrt{2}} \]

(b) Would aliasing occur if the signal \( x(t) \) is sampled at a frequency of 100Hz? What would the sampled signal \( x[n] \) be at that frequency?

No, \( 100 \text{ Hz} \not\geq 40 \text{ Hz} \)

\[ \dot{\omega} = \frac{2\pi f_s}{100} = \frac{2\pi(20)}{100} = \frac{40}{100} \pi = 0.4\pi \]

\[ x[n] = 3 \cos(\dot{\omega}n - \pi/4) \]

\[ = 3 \cos(0.4\pi n - \pi/4) \]

(c) If the \( x[n] \) in part (b) was given as input to an IDEAL D-to-C converter, what would the reconstructed signal \( x(t) \) be?

Since there is no aliasing, the reconstructed signal \( x(t) \) will be:

\[ x(t) = 3 \cos(2\pi(20) t - \pi/4) \]

which is the original signal.

(d) Would aliasing occur if the signal \( x(t) \) is sampled at a frequency of 30Hz? What would the sampled signal \( x[n] \) be at that frequency?

Yes, \( 30 \text{ Hz} < 40 \text{ Hz} \)

\[ \dot{\omega} = \frac{2\pi(30)}{30} = \frac{4\pi}{3} + 2\pi k \]

\[ x[n] = 3 \cos(\dot{\omega}n - \pi/4) \]

\[ = 3 \cos(-\frac{2\pi}{3} n - \pi/4) = 3 \cos(\frac{2\pi}{3} n + \pi/4) \]

\[ = \frac{4\pi}{3} + \frac{2\pi}{3} \]

(e) If the \( x[n] \) in part (d) was given as input to an IDEAL D-to-C converter, what would the reconstructed signal \( x(t) \) be?

Discrete \( \leftrightarrow \) Continuous

\[ \dot{x}(t) = 3 \cos\left(\frac{2\pi}{3}(30 k) + \pi/4\right) \]

\[ = 3 \cos\left(2\pi(10) k + \pi/4\right) \]
**NOTE:** when finding \( \hat{\omega} \), make sure to choose a value of \( \hat{\omega} \) such that \( -\pi < \hat{\omega} < \pi \)

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**Form C**  
EEL 3135  
Section 1471  
Name: **KEY**  
UF ID: 

**Quiz #2**

**Given**  
\[ x(t) = 4 \cos (2\pi(40)t - \pi/5) \]

(a) According to the Sampling Theorem, what should the sampling frequency be? (Give your answer in terms of the frequency of the given signal)?

\[ f_s \geq 2f_0 \]

\[ \therefore f_s \geq 2(40) \geq 80 \text{ Hz} \]

(b) Would aliasing occur if the signal \( x(t) \) is sampled at a frequency of 90Hz? What would the sampled signal \( x[n] \) be at that frequency?

\[ N_o \quad \because 90 \text{ Hz} \geq 80 \text{ Hz} \]

\[ x[n] = 4 \cos \left( \frac{\omega}{9} n - \frac{\pi}{15} \right) \]

\[ = 4 \cos \left( \frac{8\pi}{9} n - \frac{4\pi}{15} \right) \]

(c) If the \( x[n] \) in part (b) was given as input to an IDEAL D-to-C converter, what would the reconstructed signal \( x(t) \) be?

Since there was no aliasing,

\[ x(t) = 4 \cos \left( \frac{2\pi}{40} (t - \frac{\pi}{15}) \right) \]

which was the original signal.

(d) Would aliasing occur if the signal \( x(t) \) is sampled at a frequency of 50Hz? What would the sampled signal \( x[n] \) be at that frequency?

\[ \text{Yes} \quad \because 50 \text{ Hz} < 80 \text{ Hz} \]

\[ x[n] = 4 \cos \left( \frac{\omega}{f_s} n - \frac{\pi}{15} \right) \]

\[ = 4 \cos \left( -\frac{2\pi}{5} n - \frac{\pi}{15} \right) \]

\[ = 4 \cos \left( \frac{2\pi}{5} n + \frac{\pi}{15} \right) \frac{\pi}{5} \]

(e) If the \( x[n] \) in part (d) was given as input to an IDEAL D-to-C converter, what would the reconstructed signal \( x(t) \) be?

**Discrete \( \longleftrightarrow \) Continuous**

\[ n \longleftrightarrow t f_s \]

\[ x(k) = 4 \cos \left( \frac{2\pi}{5} (50 k + \pi/5) \right) = 4 \cos \left( \frac{2\pi}{5} (50 k + \pi/5) \right) \]