EEL 3135 Signals and Systems  
Fall 2001  
Professor Jian Li  
50-MINUTE EXAMINATION # 1  
September 26, 2001

There are four problems on the exam. Each problem counts 25 points. Do your work on the exam. You may use one “crib sheet” of notes written by you (Your handwriting, no xeroxing.) Hand your crib sheet in with the exam.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Score</th>
<th>Total Possible</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>total</td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

Please read and sign the following statement:

I have neither given nor received aid on this examination.

Signed:
1. (25 points) This problem concerns the signal

\[ r(t) = A \sin(\omega_0 t - \phi) \]

with \( A = 20; \quad \omega_0 = 2\pi 100; \quad \phi = -\pi/2; \)

(a) What is the period \( T_0 \) of \( r(t) \)?

(b) In the space below, sketch \( r(t) \), making certain to label \( A, \omega_0 \) and \( \phi \).

(c) Is \( r(t) \) an even function, an odd function, or neither an even nor an odd function of \( t \)?
2. (25 points) A signal is given by

\[ y(t) = 5 \cos(100\pi t + \pi/3) \]

(a) If \( y(t) \) is sampled with a sampling frequency of 10 Hz, express the sampled signal \( y[n] \) in the form \( y[n] = A \cos(\hat{\omega}_0 n + \phi) \) by finding \( A, \hat{\omega}_0, \) and \( \phi. \)

(b) What is the smallest frequency of a sinusoid that will yield the same \( y[n] \)? Justify your answer.
3. (25 points) Consider the signal

\[ q(t) = 5 \cos(2\pi 3000t) + 10 \cos(2\pi 1000t - \pi/4) \sin(2\pi 4000t - \pi/4) \]

You might find it useful to know that

\[
\sin \alpha \cos \beta = \frac{1}{2} \sin(\alpha + \beta) + \frac{1}{2} \sin(\alpha - \beta)
\]

(a) Is \( q(t) \) periodic? If so, what is the fundamental period? If not, why not?

(b) Plot the magnitude spectrum.

(c) What is the lowest sampling frequency which could be used to sample \( q(t) \) which will avoid aliasing?
4. (25 points) This problem concerns the signal $x(t)$ which is periodic with period 2.

\[
\begin{align*}
    x(t) &= 1; & 0.1 < t < 0.2 \\
    x(t) &= 0; & 0 < t \leq 0.1 \text{ and } 0.2 < t \leq 2 \\
    x(t) &= x(t + 2) & -\infty < t < \infty
\end{align*}
\]

(a) Sketch $x(t)$ for $0 < t < 4$?

(b) Compute the Fourier Series for $x(t)$. You might find it useful to know that

\[
\int e^{\alpha x} dx = \frac{1}{\alpha} e^{\alpha x}
\]