EEL 6537

Project 1 – Nonparametric Methods   Due Feb. 13, 2001

Consider the data set

$$x(n) = 2\cos(2\pi f_1 n + \phi_1) + 2\cos(2\pi f_2 n + \phi_2) + 2\cos(2\pi f_3 n + \phi_3) + z(n),$$

where $f_1 = 0.05$, $f_2 = 0.40$, $f_3 = 0.42$, and $n = 0, 1, \cdots, N-1$. The ϕ_1, ϕ_2, and ϕ_3 are independent random variables that are uniformly distributed between 0 and 2π. The noise $z(n)$, $n = 0, 1, \cdots, N-1$, is independent of ϕ_1, ϕ_2, and ϕ_3 and is obtained from

$$z(n) = -a_1 z(n-1) + u(n),$$

where $a_1 = -0.850848$. (Be sure to generate a long sequence of $z(n)$ from any initial condition and pick N samples from the steady state for each realization in your simulations. Any thoughts on generating $z(n)$ more efficiently?) The $u(0), u(1), \cdots, u(N-1)$ are independent and identically distributed real Gaussian random variables with zero-mean and variance σ^2 with $\sigma^2 = 0.101043$.

a) The True PSD:

  (a) The $z(n)$ may be considered as passing $u(n)$ through a linear time-invariance system with transfer function

$$H(\omega) = \frac{1}{1 + a_1 e^{-j\omega}}.$$

  Find and plot the PSD of $z(n)$.

  (b) Find and plot the PSD of $x(n)$ in the absence of $z(n)$.

  (c) Find and plot the PSD of $x(n)$ in the presence of $z(n)$.

b) Noiseless Case:

  (a) In the absence of $z(n)$, generate 5 realizations of $x(n)$ (using different ϕ_i, $i = 1, 2, 3$) for $N = 32$, 128, and 512. Compute the Periodogram spectral estimates and plot the 5 spectral estimates overlapped. Plot the Bartlett window that correspond to these data lengths N. For what value of N do you expect to just resolve the two closely spaced sinusoids? Why? Verify this result numerically.

  (b) Can we determine the powers of the sinusoids from the spectral estimates? Explain.

c) Noisy Case:

  (a) Generate 5 realizations of the data (using different ϕ_i, $i = 1, 2, 3$, and different noise realizations). Plot the 5 spectral estimates overlapped by using

    i. $N = 32$, Periogram.

    ii. $N = 512$, Periogram.

    iii. $N = 512$, Blackman-Tukey Method and Bartlett window with $M = 31$.

    iv. $N = 512$, Blackman-Tukey Method and Bartlett window with $M = 127$.

    v. $N = 512$, Bartlett Method with $M = 31$.

    vi. $N = 512$, Welsh Method with $M = 31$, 50% overlapping, and Bartlett window.

vii. $N = 512$, Daniell Method with $\beta = 1/31$.

(b) Discuss the results in terms of accuracy (bias and variance).

d) Write a report that details your findings. Be concise and complete.