

Consider the noiseless signal:

$$s(n) = 2 \cos(2\pi f_1 n + \phi_1) + 2 \cos(2\pi f_2 n + \phi_2) + 2 \cos(2\pi f_3 n + \phi_3),$$

where  $f_1 = 0.05$ ,  $f_2 = 0.40$ ,  $f_3 = 0.42$ , and  $n = 0, 1, \dots, N - 1$ . The  $\phi_1$ ,  $\phi_2$ , and  $\phi_3$  are independent random variables that are uniformly distributed between 0 and  $2\pi$ .

The AR noise  $z(n)$ ,  $n = 0, 1, \dots, N - 1$ , is obtained from

$$z(n) = -a_1 z(n - 1) + u(n),$$

where  $a_1 = -0.850848$ . (Be sure to use the proper initial condition). The  $u(0), u(1), \dots, u(N - 1)$  are independently and identically distributed real-valued Gaussian random variables with zero-mean and variance  $\sigma^2$  with  $\sigma^2 = 0.101043$  and are independent of  $\phi_1$ ,  $\phi_2$ , and  $\phi_3$ .

- a) **AR Noise Case:** Generate 5 realizations of the AR noise  $z(n)$  for  $N = 16$ .
- Compute the Periodogram and plot the 5 spectral estimates overlapped.
  - Assume an AR( $p$ ) model, where  $p = 1, 3$ . Use the Yule-Walker approach with the biased autocorrelation estimates and the F/B Prony approach. Plot the 5 spectral estimates overlapped.
  - Compare the spectral estimates with the true PSD and discuss the results.
  - Apply different order selection methods to the data and discuss the observations.
- b) **AR Noise and Signal Case:** Let

$$x(n) = s(n) + z(n).$$

Generate 5 realizations of the data for  $N = 64$ .

- Assume an AR(6) model. Plot the 5 spectral estimates overlapped by using the F/B Prony approach.
  - Discuss and compare the sinusoidal frequency estimates obtained from the peaks of the spectral estimates and comment.
- c) **White Noise and Signal Case:** Let

$$x(n) = s(n) + u(n).$$

Generate 5 realizations of  $x(n)$  for  $N = 16$  and  $N = 64$ .

- Assume that about one third of the data in the middle is missing for  $N = 16$ . Use whatever methods and their variations you deem necessary to estimate the missing data as well as the power spectrum of the original data. Justify and comment on the effectiveness of your approaches.
  - For  $N = 64$ , we also have one third of the data missing. These missing data occur in one or more gaps. Comment on the impact of gap locations and sizes on the problem of estimating the missing data and the power spectrum. Devise approaches for the estimation problem and comment on their effectiveness.
- d) Write a report that details your findings. Be concise and complete.