High Resolution Angle-Doppler Imaging for MTI Radar

Jian Li\textsuperscript{2*}  Xumin Zhu\textsuperscript{3}  Petre Stoica\textsuperscript{4}  Muralidhar Rangaswamy\textsuperscript{5}

Abstract

To reduce the need for training data or for accurate prior knowledge of the clutter statistics in space-time adaptive processing (STAP), we consider high resolution angle-Doppler imaging by processing each range bin of interest independently. Specifically, we use a weighted least squares based iterative adaptive approach (IAA) to form angle-Doppler images of both clutter and targets for each range bin of interest. The resulting angle-Doppler images can be used with localized detection approaches for moving target indication (MTI). We show via numerical examples that the robust and non-parametric IAA algorithm can be used to enhance the MTI performance significantly as compared to existing approaches.

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I. INTRODUCTION

In the conventional STAP, the clutter-and-noise covariance matrix of the range bin of current interest, let us call it $R_{CN}$, is estimated from training data (presumed to be target free and homogeneous). Given, say $N$ adjacent range bin snapshots, denoted as $\{z(n)\}_{n=1}^{N}$, $R_{CN}$ is estimated by means of a well-known formula for the sample covariance matrix (see, e.g., [1]–[5]):

$$\hat{R}_{CN} = \frac{1}{N} \sum_{n=1}^{N} z(n)z^*(n), \tag{1}$$

where $(\cdot)^*$ denotes the conjugate transpose. However, frequently the dimension of $R_{CN}$ (denoted by $M$ in what follows) is larger than $N$ as $N$ has to be kept small due to the inhomogeneous nature of the clutter and the fact that the adjacent range bins are not necessarily target free. The result is that $\hat{R}_{CN}$ is, more often than not, a poor estimate of $R_{CN}$. This is particularly so when $M \gg N$, resulting in a rank-deficient $\hat{R}_{CN}$. Many approaches have been proposed for selecting high quality training data and for avoiding the rank-deficient problem of $\hat{R}_{CN}$ (see, e.g., [1]–[3], [6]). However, in the presence of multiple targets, the angle-Doppler image formed by using $\hat{R}_{CN}$, no matter how accurate $\hat{R}_{CN}$ is, is not optimal in any sense since the target statistics are not accounted for in $\hat{R}_{CN}$.

Getting high quality training data has turned out to be a challenging problem. As a result, knowledge-aided STAP has been attracting attention lately (see, e.g., [4], [7]–[15]). However, getting accurate prior knowledge on the clutter statistics can be rather expensive. The prior knowledge may also be inaccurate due to environmental changes or outdated intelligence information. Using inaccurate prior knowledge can degrade rather than improve the STAP performance (see, e.g., [7]).

To reduce the need for training data or for accurate prior knowledge of the clutter statistics, many approaches have been considered in the literature (see, e.g., [16]–[26]). The joint-domain localized approach proposed in [16] requires using the delay-and-sum (DAS) (i.e., least-squares or matched filter) type of approaches to transform the data into the angle-Doppler domain. (DAS becomes the discrete Fourier transform for the case of uniform linear arrays (ULAs) and constant pulse repetition frequencies (PRFs) [16].) It is well-known, however, that the angle-Doppler images formed by such data-independent
approaches suffer from broad main-beam (smearing) and high sidelobe level (leakage) problems. For the case of ULAs and constant PRFs, one can form multiple “snapshots” by taking sub-apertures in both space and time (see, e.g., [17]–[19]). However, this is done at the cost of reduced resolution. Moreover, in practice, the arrays may not be uniform and linear. Even for arrays intended to be uniform and linear, due to the presence of mutual coupling and other array artifacts, they can end up not being so [13]. The parametric approaches considered in [21]–[23], [25], [26] model the clutter and noise as a vector autoregressive (VAR) random process. The parametric approaches are known to perform better than their non-parametric counterparts if the assumed parametric data model is accurate. However, the parametric approaches tend to be sensitive to model errors, which are inevitable in practice. A variation of the CLEAN algorithm is considered in [24], and a global matched filter approach is presented in [20] for STAP applications. Both methods can be used for high resolution angle-Doppler imaging of both clutter and targets for each range bin of interest independently, i.e., using only the data from the range bin of interest (which is the so-called primary data). They belong to the class of sparse signal representation methods, which have been attracting attention in recent years [27]- [40]. However, the CLEAN type of algorithms assumes point scatterers while the clutter ridge due to stationary ground leads typically to a continuous spectrum. As a result, a large number of point scatterers is needed to approximate the clutter ridge [24]. Moreover, the CLEAN algorithm yields biased estimates for closely-spaced point scatterers [41], [42]. The global matched filter algorithm usually requires large computation times and the tuning of one or more user parameters, which may limit its practicality.

We also consider high resolution angle-Doppler imaging by processing each range bin of interest independently. We use a weighted least-squares based iterative adaptive approach (IAA) [42]–[45]. IAA is a robust and nonparametric adaptive algorithm that can be used for angle-Doppler imaging of both clutter and targets based on the primary data only. IAA can work with arbitrary array geometries and random slow-time samples. The high resolution angle-Doppler images formed by IAA can be used with localized detection approaches for moving target indication (MTI).
This paper is organized as follows. In Section II, we describe the IAA-based high resolution angle-Doppler imaging approach. In Section III, we provide numerical examples comparing the performances of IAA and various covariance matrix inversion based angle-Doppler imaging approaches. We also demonstrate the target detection performances when the high resolution angle-Doppler images are used with a simple median detector for MTI. Finally, Section IV concludes this paper.

II. ANGLE-DOPPLER IMAGING VIA IAA

IAA [42] can be used to form an angle-Doppler image for each range bin of interest (ROI) using only the primary data. Assume that the radar system has \( L \) antennas forming an arbitrary linear array and that it transmits \( P \) pulses during a coherent processing interval (CPI). Let \( M = PL \). Within the CPI, we assume that echoes from \( I \) range bins are collected by the radar. For a fixed elevation angle, a target can be specified by its range index \( i \), azimuth angle (or spatial frequency \( \omega_s \)), and Doppler frequency \( \omega_D \). Its “nominal” space-time steering vector \( \mathbf{a}(\omega_s, \omega_D) \in \mathbb{C}^{M \times 1} \) can be expressed as follows:

\[
\mathbf{a}(\omega_s, \omega_D) = \tilde{\mathbf{a}}(\omega_D) \otimes \mathbf{\bar{a}}(\omega_s),
\]

where \( \otimes \) denotes the Kronecker product, and \( \tilde{\mathbf{a}}(\omega_s) \) and \( \mathbf{\bar{a}}(\omega_D) \), respectively, denote the spatial and temporal (slow-time) steering vectors.

For each ROI, we scan over both angle and Doppler dimensions to form its angle-Doppler image, i.e., to compute the two-dimensional power distribution of targets as well as clutter-and-noise, using the primary data only. For notational convenience, we drop below the dependence on the range bin index. Assume that the number of angular and Doppler scanning (grid) points are \( K \) and \( \tilde{K} \), respectively, which determine the smoothness of the angle-Doppler image formed by IAA. (Usually \( K \) should be chosen from 5\( L \) to 10\( L \) and \( \tilde{K} \) from 5\( P \) to 10\( P \).) Then the total number of scanning points is \( K = \tilde{K} \tilde{K} \). Let \( \mathbf{P} \) be a diagonal matrix of dimension \( K \) with the powers corresponding to the scanning points on the diagonal. Given \( \mathbf{P} \), we can construct the following IAA covariance matrix for the ROI:

\[
\mathbf{R}_{\text{IAA}} = \mathbf{A} \mathbf{P} \mathbf{A}^*,
\]
where \( A = [a(\omega_{S_1}, \omega_{D_1}), a(\omega_{S_1}, \omega_{D_2}), \ldots, a(\omega_{S_K}, \omega_{D_K})] \) is an \( M \times K \) steering matrix. Given \( R_{\text{IAA}} \) in (3) and also the primary data vector \( y \) for the ROI, an estimate of the power \( P_{\tilde{k} \tilde{k}} \), denoted as \( \hat{P}_{\tilde{k} \tilde{k}} \), at the scanning point \( (\omega_{S_\tilde{k}}, \omega_{D_\tilde{k}}) \), can be computed as:

\[
\hat{P}_{\tilde{k} \tilde{k}} = \left| \frac{a^*(\omega_{S_\tilde{k}}, \omega_{D_\tilde{k}})R_{\text{IAA}}^{-1}y}{a^*(\omega_{S_k}, \omega_{D_k})R_{\text{IAA}}^{-1}a(\omega_{S_k}, \omega_{D_k})} \right|^2, \tag{4}
\]

where \( P_{\tilde{k} \tilde{k}} \) is a diagonal element of \( P \), and \( | \cdot | \) denotes the absolute value. (Note that (4) can be obtained via whitening using \( R_{\text{IAA}} \) followed by matched filtering.) Since IAA requires \( R_{\text{IAA}} \), which depends on the unknown powers, it must be implemented as an iterative approach. The initialization is done by the standard DAS beamformer, i.e., the so-called matched filter, for which the signal power is determined in the same way as for IAA except that \( R_{\text{IAA}} \) in (4) is replaced by the identity matrix \( I \). The IAA algorithm is summarized in Table 1. The iterative process stops when a prescribed iteration number is achieved. This number is set to 10 in our simulations as we have observed no obvious performance improvement beyond 10 iterations.

**TABLE I**

<table>
<thead>
<tr>
<th>THE IAA ALGORITHM</th>
</tr>
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<tbody>
<tr>
<td><strong>initialize</strong> ( P_{k,k} = \frac{1}{M^2}</td>
</tr>
<tr>
<td><strong>repeat</strong> ( R_{\text{IAA}} = APA^* )</td>
</tr>
<tr>
<td>for ( \tilde{k} = 1, \ldots, \bar{K} )</td>
</tr>
<tr>
<td>for ( k = 1, \ldots, \bar{K} )</td>
</tr>
<tr>
<td>( P_{k,k} = \left</td>
</tr>
<tr>
<td><strong>end</strong></td>
</tr>
<tr>
<td><strong>end</strong></td>
</tr>
<tr>
<td><strong>until</strong> a certain number of iterations is reached</td>
</tr>
</tbody>
</table>

The computational complexity of IAA is on the order of \( O(M^2K) \), where we remind the reader that \( K \gg M \) is the number of grid points in the angle-Doppler image. The computational complexity of IAA can be significantly reduced for the case of ULAs and constant PRFs by exploiting the Toeplitz-block-Toeplitz structure of \( R_{\text{IAA}} \) [46].
III. PERFORMANCE STUDIES

Our numerical examples are either based on the KASSPER data [47] or on data simulated by ourselves. Our simulations are based on the KASSPER setup, which simulates both realistic inhomogeneous clutter and intrinsic clutter motion (ICM).

Consider first the data that we simulated. We simulate an airborne radar system with \( P = 32 \) pulses and \( L = 11 \) spatial channels, yielding \( M = PL = 352 \) degrees-of-freedom (DOFs). The platform is heading toward west with a speed of 100 m/s. The main-beam of the radar is steered toward an azimuth angle of 195° measured clockwise from the true north and an elevation of -5° relative to the horizon. The radar pulse repetition frequency (PRF) is 1984 Hz. For each CPI, a total of \( I = 1000 \) range bins are sampled covering a range swath of interest from 35 km to 50 km. Calibration errors, such as angle-independent phase errors and angle-dependent subarray position errors, can also be included in our simulated data, making the assumed steering vector used for angle-Doppler imaging different from the true one. In the numerical examples, we consider both circumstances with and without steering vector errors. Since \( \mathbf{R}_{cn}, \tilde{\mathbf{R}}_{cn}, \) and \( \mathbf{R}_{tia} \) all vary with the range bin index \( i \), in what follows, we will indicate explicitly the dependence of these covariance matrices on the range bin index for the sake of clarity. We generate the clutter-and-noise data for the \( i \)th range bin as:

\[
\mathbf{e}_i = \mathbf{R}_{cn}^{1/2}(i)\mathbf{v}_i, \quad i = 1, \ldots, I, \tag{5}
\]

where \((\cdot)^{1/2}\) denotes a Hermitian matrix square root and \(\{\mathbf{v}_i\} \in \mathbb{C}^{M \times 1}\) are independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian random vectors with mean 0 and covariance matrix I.

A. Angle-Doppler Imaging

Consider first angle-Doppler imaging of the clutter-and-noise only. Let \( \tilde{\mathbf{R}}(i) \) denote the covariance matrix used to form the angle-Doppler image of the \( i \)th range bin. The power estimate of the clutter-and-noise at a given angle and Doppler pair \((\omega_s, \omega_d)\) is computed, similarly to (4), as follows:
Consider first the case where the ROI has range bin index \( i = 100 \) and the steering vector errors are absent, i.e., the steering vector \( a \) used in (6) above is the same as the true one. The angle-Doppler image obtained by using the true clutter-and-noise covariance matrix \( R_{CN}(i) \) in lieu of \( \tilde{R}(i) \) in (6) is shown in Figure 1(a). (The true covariance matrices are always formed using the true steering vectors.) For a side-looking airborne radar and small crab angle, it is well-known that the clutter Doppler frequency depends linearly on the sinusoidal value of the azimuth angle. Thus, the clutter must lie on the diagonal “clutter ridge”, as confirmed in Figure 1(a). Note that (6) with \( \tilde{R}(i) = R_{CN}(i) \) becomes the standard Capon beamformer (SCB) [48] (assuming that \( R_{CN}(i) \) is available) and the data-adaptive SCB is known to provide high resolution and low sidelobe levels, as can been observed from Figure 1(a). The angle-Doppler image of the clutter-and-noise obtained via the data-independent DAS beamformer [i.e., using \( \tilde{R}(i) = I \) in (6)] is shown in Figure 1(b). The smearing and leakage problems of DAS are obvious from Figure 1(b). The IAA image, see Figure 1(c), is obtained by using a uniform angular scanning grid for the azimuth angle ranging from \( 90^\circ \) to \( 270^\circ \) with a \( 2^\circ \) grid step, i.e., \( \bar{K} = 90 \), and also a uniform Doppler scanning grid for the Doppler frequency ranging from \(-\pi\) to \( \pi\) with \( \tilde{K} = 256 \). Note that the IAA image has much better resolution and much lower sidelobe level than that of DAS and that the IAA estimated clutter power is well focused along the diagonal clutter ridge.

Consider again the above example but now in the presence of steering vector errors, i.e., the assumed steering vector \( a \) used for angle-Doppler imaging in (6) is different from the true one. The steering vector errors are the same as those present in the KASSPER data [47]. Figure 1(d) displays the IAA image for this case. Note that IAA is quite robust against the presence of steering vector errors and that the diagonal clutter ridge can still be observed clearly, though there is some smearing away from the clutter ridge. The robustness of IAA is due to the fact that the steering vectors used to form \( R_{IAA}(i) \) are not the true ones, but the assumed ones. As a result, unlike SCB, IAA will not suffer from significant signal cancellation.
problems [48], although IAA does suffer some performance degradation, as can be seen by comparing Figures 1(c) and (d). The robustness of DAS and the sensitivity of SCB to steering vector errors will be illustrated in the following examples.

Consider next the angle-Doppler imaging performance in the presence of targets. We insert a total of $K_0 = 200$ targets spread over the entire range-Doppler map at the azimuth angle of 195°. Each target is assumed to have a constant power $\sigma_0^2$, as shown in Figure 2, where the ground truth is denoted by “o”. The targets have an average signal-to-clutter-and-noise ratio (SCNR) of -18.9 dB, where the average SCNR is defined as:

$$
\frac{1}{K_0} \sum_{k=1}^{K_0} \frac{\text{tr} \left[ \sigma_0^2 a_0(\omega_{S0}, \omega_{Dk}) a_0^*(\omega_{S0}, \omega_{Dk}) \right]}{\text{tr} \left[ R_{CN}(i_k) \right]}. \quad (7)
$$

In (7), $a_0(\omega_{S0}, \omega_{Dk})$ is the true steering vector corresponding to the fixed spatial frequency $\omega_{S0}$ for the 195° azimuth angle and the Doppler frequency $\omega_{Dk}$ for the $k$th target at range bin $i_k$.

The power estimate obtained from the received signal $y_i$, which consists of both targets and clutter-and-noise, at a given angle and Doppler pair $(\omega_s, \omega_d)$ is computed similarly to (6) except that $e_i$ is now replaced by $y_i$.

We compare the performance achieved by using the covariance matrix $R_{IAA}(i)$ to the performance corresponding to various alternative covariance matrices, namely: the true clutter-and-noise covariance matrix $R_{cn}(i)$, the true target-clutter-and-noise covariance matrix $R_{tcn}(i)$, and an imprecise prior knowledge-based covariance matrix $R_0(i)$. For the clairvoyant case of known $R_{tcn}(i)$, $R_{tcn}(i)$ is given by:

$$
R_{tcn}(i) = R_{cn}(i) + \sum_{k=1}^{K_0(i)} \sigma_0^2 a_0(\omega_{S0}, \omega_{Dk}(i)) a_0^*(\omega_{S0}, \omega_{Dk}(i)), \quad (8)
$$

where $K_0(i)$ denotes the number of targets for the $i$th range bin and $\omega_{Dk}(i)$ denotes the Doppler frequency of the $k$th target at the $i$th range bin. In our simulations, $R_0(i)$ is constructed as a perturbed version of $R_{cn}(i)$ [7]:

$$
R_0(i) = R_{cn}(i) \odot t_i t_i^*, \quad (9)
$$
where ⊙ denotes the Hadamard matrix product, and \( \mathbf{t}_i \) is a vector of i.i.d. complex Gaussian random variables with mean 1 and variance \( \sigma_t^2 = 0.1 \).

For the sake of clarity, we first examine the power estimation performance at the fixed azimuth angle of 195° and the ROI with range bin index \( i = 66 \). The estimated power distribution as a function of the Doppler frequency without steering vector errors is shown in Figure 3. Figures 3(a), 3(b) and 3(c) are obtained by using \( \mathbf{R}_{\text{TCN}}(i) \), \( \mathbf{R}_{\text{CN}}(i) \) and \( \mathbf{R}_{\text{IAA}}(i) \) in lieu of \( \mathbf{R}(i) \) in (6), respectively. The solid vertical lines indicate the locations of the two targets at this range bin. Note from Figure 3(b) the poor target resolution and high sidelobe level problems associated with using the true clutter-and-noise covariance matrix \( \mathbf{R}_{\text{CN}}(i) \), even in the absence of steering vector errors. This result occurs because \( \mathbf{R}_{\text{CN}}(i) \) does not contain target information and hence the adaptive processing is not adapted to the presence of targets. Therefore, the power estimation using \( \mathbf{R}_{\text{CN}}(i) \) in general is not optimal. (The only optimal case is when there is a single target in the ROI and the steering vector is pointed precisely at the target location.) Moreover, as we have already mentioned, the operation of

\[
\left| \frac{\mathbf{a}^*(\omega_S, \omega_D) \mathbf{R}_{\text{CN}}^{-1}(i) \mathbf{y}_i}{\mathbf{a}^*(\omega_S, \omega_D) \mathbf{R}_{\text{CN}}^{-1}(i) \mathbf{a}(\omega_S, \omega_D)} \right|^2
\]

is basically whitening via using \( \mathbf{R}_{\text{CN}}^{-1/2}(i) \), followed by matched filtering, resulting in target resolutions and sidelobe levels similar to or perhaps even worse than those of the DAS type of approaches.

Figure 4 is the same as Figure 3 except that now steering vector errors exist. As expected, in the presence of steering vector errors, the power estimates of both targets and clutter obtained by using the true \( \mathbf{R}_{\text{TCN}}(i) \) are much worse than those obtained in the absence of array steering vectors due to the well-known sensitivity of SCB to steering vector errors. In the presence of even slight steering vector errors, SCB tends to suppress the desired signal as if it were an interference, causing signal cancellation problems [48]. Also as expected, the clutter power estimate obtained by using the true \( \mathbf{R}_{\text{CN}}(i) \) is severely under estimated, whereas the target power estimates are not sensitive to the steering vector errors, due to the target not being contained in \( \mathbf{R}_{\text{CN}}(i) \) and hence not being suppressed by adaptive processing. However, as we will see later on in Figure 8, this somewhat desirable behavior does not result in improved target detection performance. We observe from Figures 3(c) and 4(c) that the power estimates obtained by IAA
exhibit much sharper peaks around the true target locations and the sidelobe levels are also much lower as compared to those in Figures 3(b) and 4(b) with or without steering vector errors. The robustness of IAA is again due to the fact that the steering vectors used to form $R_{\text{IAA}}(i)$ are not the true ones, but the assumed ones, and using the same assumed steering vectors with $R_{\text{IAA}}(i)$ for angle-Doppler imaging will not result in signal cancellation problems. IAA does suffer, though, from some performance degradation in the presence of steering vector errors, but the severity is more like that of the degradation suffered by DAS.

We now compare the angle-Doppler images formed using IAA and other methods for the ROI with range bin index $i = 66$. Figures 5 and 6, respectively, are for the cases of without and with array steering vector errors. Figures 5(a) and 6(a) are obtained by using $R_{\text{TCN}}(i)$. Note that in the absence of steering vector errors, the angle-Doppler image formed by using $R_{\text{TCN}}(i)$ is very sharp, with the clutter well focused along the diagonal ridge and the two moving targets clearly visible. In the presence of steering vector errors, however, the angle-Doppler image formed by using $R_{\text{TCN}}(i)$ is much worse, due to the signal cancellation problems of SCB.

Figures 5(b) and 6(b) are obtained by using the prior knowledge-based clutter-and-noise covariance matrix, $R_0(i)$, which is a perturbed version of the true clutter-and-noise covariance matrix $R_{\text{CN}}(i)$. Note that the angle-Doppler images formed by using $R_0(i)$ are rather smeared and of poor quality. Figures 5(c) and 6(c) are obtained by using $R_{\text{CN}}(i)$. Note the obvious smearing caused by the presence of the moving targets. Figures 5(d) and 6(d) are generated by using the DAS approach. Due to the smearing and leakage problems of DAS, the two moving targets are barely visible.

Figures 5(e) and 6(e) are obtained by using IAA. Note that the two moving targets are clearly visible in both cases. In the absence of steering vector errors, the IAA image is close to the clairvoyant image formed using $R_{\text{TCN}}(i)$. In the presence of steering vector errors, however, the angle-Doppler image formed by IAA is better than the clairvoyant image formed using $R_{\text{TCN}}(i)$, due to the robustness of IAA against steering vector errors.
For comparison purposes, we show in Figure 5(f) the angle-Doppler image obtained by using the so-called Signal and Clutter as Highly Independent Structured Modes (SCHISM) algorithm proposed in [24]. Note that to determine the spatial and Doppler frequency parameters for each mode, nonlinear operations are required for this algorithm, which increases the computational load, especially when the number of modes is large. A 30 dB Taylor space-time taper is applied and a total number of 60 modes are kept for this specific range bin (due to the high computational demand of SCHISM, we stopped at 60 modes). As we can see from Figure 5(f), strong discrete scatterers are found along the clutter ridge. However, only one of the targets is found by SCHISM, and the estimate of the target amplitude is quite inaccurate, making it barely visible.

The ICM level considered in the KASSPER data set is moderate. To illustrate the effects of severe ICM on the performance of IAA, we increase the ICM level by applying a matrix taper to the clutter-and-noise covariance matrix [47]. Figures 7(a) and 7(b) show the corresponding imaging results, for range bin 66 in the absence of steering vector errors, obtained by using $R_{TCN}(i)$ and $R_{IAA}(i)$, respectively. As we can see, due to the higher level of ICM, the diagonal clutter ridge is much broadened. However, as shown in Figure 7(b), similar to the clairvoyant case of known $R_{TCN}(i)$, IAA can still resolve the two targets, showing robustness to the presence of ICM.

**B. Target Detection**

The high quality angle-Doppler images formed by IAA can be combined with localized detection approaches as well as other target tracking approaches for target detection. Below, we only consider using a simple detector for illustration purposes. The full potential offered by exploiting the angle-Doppler images formed by IAA should be investigated further, using more extensive simulated as well as measured data. This, however, is beyond the scope of the current paper.

We consider target detection based on the angle-Doppler images generated by using various covariance matrices with $\omega_S$ fixed to $\omega_{S_0}$ corresponding to the 195° azimuth angle. To distinguish between targets and clutter to avoid false alarms, one might think of discarding the peaks that are close to the diagonal
clutter ridge. However, this would require prior knowledge on operating parameters of the radar, and also there is no clear guidance as to how to determine the “width” of the ridge. Another way, which will be used here, is to rely on the assumption that for the fixed angle and a given Doppler bin, the clutter peaks will be nearly the same in a few (say, 10) range bins that are adjacent to each ROI, whereas the target peaks are not so “dense” in range. We use a median constant false alarm (CFAR) detector, which has the following form [49]:

$$10 \log_{10} \left| \frac{a^*(\omega_s, \omega_D) \tilde{R}^{-1}(i)y_i}{a^*(\omega_s, \omega_D) \tilde{R}^{-1}(i)a(\omega_s, \omega_D)} \right|^2 - 10 \log_{10} \eta(i, \omega_{S0}, \omega_D) H_1 \geq H_0 \xi,$$

(10)

where $H_0$ is the null hypothesis (i.e., no target), $H_1$ is the alternative hypothesis (i.e., $H_0$ is false) and $\xi$ is a target detection threshold. The background clutter-and-noise level $\eta(i, \omega_{S0}, \omega_D)$ for range bin $i$, spatial frequency $\omega_{S0}$, and Doppler frequency $\omega_D$ is estimated as the median value of the set of power levels from 10 adjacent range bins at $(\omega_{S0}, \omega_D)$. For each threshold $\xi$, the number of correct target detections as well as the number of false alarms are recorded to yield the receiver operating characteristic (ROC) [i.e., the probability of detection (PD) versus the probability of false alarm (PFA)] curves. In our simulations, the $k$th target with Doppler frequency $\omega_{Dk}$ is considered to be detected correctly if there are any number of detections in the $i_k$th range bin falling within the interval $(\omega_{Dk} - \pi/32, \omega_{Dk} + \pi/32)$. We remark that the median CFAR detector does not use the data from the adjacent range bins in the same way as the conventional STAP approaches do since the adjacent range bins are used by the median detector after high resolution angle-Doppler imaging and are for local comparison of power levels only. The conventional STAP approaches use the training data for space-time adaptive processing.

We consider below the cases with and without steering vectors. The steering vector error occurs when the assumed steering vector $a(\omega_s, \omega_D)$ used in (10) is different from the true one. The true covariance matrices $R_{\text{TCN}}(i)$ and $R_{\text{CN}}(i)$ are always formed with the true steering vectors.

In Figure 8, we show the ROC curves of the IAA-based median detector [(10) with $\tilde{R}(i)$ replaced by $R_{\text{IAA}}(i)$]. For comparison purposes, we also show the ROC curves corresponding to the detectors using $R_{\text{TCN}}(i)$ [(10) with $\tilde{R}(i)$ replaced by $R_{\text{TCN}}(i)$], $R_{\text{CN}}(i)$ [(10) with $\tilde{R}(i)$ replaced by $R_{\text{CN}}(i)$], and also a
perturbed $R_{cn}(i)$ [(10) with $\tilde{R}(i)$ replaced by $R_0(i)$]. Figures 8(a) and 8(b) are for the cases without and with steering vector errors, respectively. As we can see, in the absence of steering vector errors, the detection performance of using the angle-Doppler images obtained by IAA almost coincides with that of the clairvoyant detector using $R_{tcn}(i)$. In the presence of steering vector errors, however, using the angle-Doppler images obtained by IAA outperforms even the clairvoyant detector using $R_{tcn}(i)$. This is not surprising because SCB is sensitive to array steering vector errors whereas IAA is robust against such errors. Note also that using the angle-Doppler images obtained by IAA outperforms the detector using $R_{cn}(i)$.

Next, we compare the target detection performance of IAA-based median detector (i.e., using the angle-Doppler images obtained by IAA with the aforementioned median detector) with that of the adaptive matched filter (AMF) detector [50]. The AMF detector has the form:

$$\left|a^*(\omega_s, \omega_d)\tilde{R}^{-1}(i)y_i\right|^2 \frac{H_1}{H_0} \gtrapprox \xi_{AMF},$$

where $\xi_{AMF}$ is a target detection threshold.

We consider the AMF detector, where the $\tilde{R}(i)$ in (11) is replaced by $R_{cn}(i)$. The angle-Doppler images formed by using the left side of (11), where the $\tilde{R}(i)$ in (11) is replaced by $R_{cn}(i)$, for the $i = 66$th range bin are shown in Figures 9(a) and 10(a), respectively, for the cases without and with array steering vectors. Note that the clutter ridge is suppressed in the angle-Doppler images formed by using the ideal AMF. The quality of the angle-Doppler images formed by using the ideal AMF is about the same with or without steering vector errors due to the target information not being included in $R_{cn}(i)$. The resolution of the angle-Doppler images formed by using the ideal AMF, however, is much poorer than that of the IAA images shown in Figures 5(e) and 6(e). The ROC curves obtained in the cases without and with steering vector errors are shown in Figures 11(a) and 11(b), respectively. Note that the IAA-based median detector significantly outperforms the ideal AMF detector.

We also consider the sample matrix inversion (SMI) based AMF detector where the $\tilde{R}(i)$ in (11) is replaced by the sample clutter-and-noise covariance matrix $\hat{R}_{cn}(i)$ estimated from $N = 2M = 704$ training
data (i.e., the clutter-and-noise only data from the adjacent range bins). Note that we are considering here the best scenario for the SMI based AMF by assuming that the targets in the training data were somehow magically removed. The angle-Doppler images formed by using the left side of (11), where the $\hat{R}(i)$ in (11) is replaced by the $\hat{R}_{\text{cn}}(i)$, for the $i = 66$th range bin are shown in Figures 9(b) and 10(b), respectively, for the cases without and with array steering vector errors. Note that the angle-Doppler images formed by using the SMI based AMF have a much noisier background than those formed by using the ideal AMF. The quality of the angle-Doppler images formed by using the SMI based AMF is about the same with or without steering vector errors, again due to the target information not being included in $\hat{R}_{\text{cn}}(i)$. The resolution of the angle-Doppler images formed by using the SMI based AMF is about the same as that of the images formed by using the ideal AMF. The ROC curves obtained by using the SMI based AMF for the cases without and with steering vector errors are shown in Figures 11(a) and 11(b), respectively. For both cases, the SMI based AMF performs only slightly worse than the ideal AMF, even though the angle-Doppler images formed by the former have a much noisier background than those formed by the latter. This result occurs because the targets are much stronger than the background in the images formed by both detectors. Note again that the IAA-based median detector significantly outperforms the AMF detectors.

Finally, we evaluate the performance of IAA using the KASSPER data [47]. In addition to the inhomogeneous clutter (with $R(i)$ varying with range bin $i$), the KASSPER data also include many other real-world efforts, such as subspace leakage, array calibration errors (and hence steering vector errors), and multiple ground targets. Moreover, some of the targets have rather weak power levels and some are very slowly moving, which makes the KASSPER data more challenging than the data we simulated.

The radar main-beam for the KASSPER data has a width of $10^\circ$. The radar attempts to detect targets in the azimuth range of $[190^\circ, 200^\circ]$ instead of a fixed azimuth angle of $195^\circ$. Therefore, in addition to range and Doppler, the azimuth angle is treated as another dimension (in our simulated data, we fixed the azimuth angle at $195^\circ$). Given the spatial and Doppler frequency pair $(\omega_{S_k}, \omega_{D_k})$ of the $k$th target,
the target is considered to be detected if there are any number of detections in the \( i_k \)th range bin falling within the area of \((\theta_k - 5^\circ, \theta_k + 5^\circ)\) and \((\omega_{D_k} - \pi/32, \omega_{D_k} + \pi/32)\), where \( \theta_k \) denotes the azimuth angle of the \( k \)th target. The corresponding ROC curves are shown in Figure 12. (Note that the target power information used to generate the KASSPER data is not available to us. Therefore, \( R_{\text{CN}}(i) \) is unknown and hence the corresponding ROC curve is not shown in Figure 12.) Again, IAA gives the best performance and outperforms even the detector using \( R_{\text{CN}}(i) \), which is rarely available in practical applications.

In Figure 13, we compare the ROC curves corresponding to the IAA-based median detector and the AMF detector using \( R_{\text{CN}} \). Note that the clutter-only data is not available for the KASSPER set, and hence the AMF detector using \( \hat{R}_{\text{CN}} \) is not considered here. Again, as shown in Figure 13, IAA gives a much better performance than the ideal AMF detector.

IV. CONCLUSIONS

The conventional space-time adaptive processing (STAP) approaches require the use of training data from adjacent range bins to obtain an estimate of the clutter-and-noise covariance matrix. This estimate is used to whiten the clutter-and-noise statistics, an operation that is followed by matched filtering for angle-Doppler imaging. Due to the often poor quality of the estimate of the clutter-and-noise covariance matrix as well as the poor target resolution and high sidelobe problems of matched filtering, the performance of the conventional STAP approaches can be unacceptable, especially when the clutter is severely inhomogeneous and the targets are slowly moving. We have presented herein a nonparametric iterative adaptive approach (IAA) to angle-Doppler imaging for airborne surveillance radar systems. Due to adapting to both clutter and targets, the angle-Doppler images formed via IAA have much higher resolution and much lower sidelobe levels compared to the conventional approaches. We have used numerical examples to demonstrate the usefulness of IAA to forming high quality angle-Doppler images followed by using a simple localized detector for enhanced MTI performance.
REFERENCES


Fig. 1. Angle-Doppler images of the clutter and noise obtained by using (a) the true clutter-and-noise covariance matrix $R_{CN}(i)$ without steering vector errors, (b) DAS without steering vector errors, (c) IAA without steering vector errors, and (d) IAA with steering vector errors.
Fig. 2. Ground truth of targets.

Fig. 3. Power distribution over Doppler at the azimuth angle of 195° for the \(i = 66\)th range bin obtained in the absence of steering vector errors by using (a) the true target-clutter-and-noise covariance matrix \(R_{TCN(i)}\), (b) the true clutter-and-noise covariance matrix \(R_{CN(i)}\), and (c) IAA.

Fig. 4. Power distribution over Doppler at the azimuth angle of 195° for the \(i = 66\)th range bin obtained in the presence of steering vector errors by using (a) the true target-clutter-and-noise covariance matrix \(R_{TCN(i)}\), (b) the true clutter-and-noise covariance matrix \(R_{CN(i)}\), and (c) IAA.
Fig. 5. Angle-Doppler images for the $i = 66$th range bin obtained in the absence of steering vector errors by using (a) the true target-clutter-and-noise covariance matrix $R_{TCN}(i)$, (b) the imprecise clutter-and-noise covariance matrix $R_0(i)$, (c) the true clutter-and-noise covariance matrix $R_{CN}(i)$, (d) DAS, (e) IAA, and (f) SCHISM. The two circles indicate the true locations of the two moving targets.
Fig. 6. Angle-Doppler images for the $i = 66$th range bin obtained in the presence of steering vector errors by using (a) the true target-clutter-and-noise covariance matrix $R_{TCN}(i)$, (b) the imprecise clutter-and-noise covariance matrix $R_0(i)$, (c) the true clutter-and-noise covariance matrix $R_{CN}(i)$, (d) DAS, and (e) IAA. The two circles indicate the true locations of the two moving targets.
Fig. 7. Angle-Doppler images for the $i = 66$th range bin obtained in the absence of steering vector errors with increased ICM level by using (a) the true target-clutter-and-noise covariance matrix $R_{TCN}(i)$, (b) IAA. The two circles indicate the true locations of the two moving targets.

Fig. 8. ROC curves for the data we simulated, based on the KASSPER setup: (a) without steering vector errors, and (b) with steering vectors errors.

Fig. 9. The AMF angle-Doppler images for the $i = 66$th range bin obtained in the absence of steering vector errors by using (a) the true clutter-and-noise covariance matrix $R_{CN}(i)$, and (b) the estimate $\hat{R}_{CN}(i)$. The two circles indicate the true locations of the two moving targets.
Fig. 10. The AMF angle-Doppler images for the $i = 66$th range bin obtained in the presence of steering vector errors by using (a) the true clutter-and-noise covariance matrix $R_{CN}(i)$, and (b) the estimate $\hat{R}_{CN}(i)$. The two circles indicate the true locations of the two moving targets.

Fig. 11. ROC curves for the data we simulated when using the IAA-based median detector and the AMF detectors with $R_{CN}$ and its estimate $\hat{R}_{CN}$, for the cases: (a) without steering vector errors and (b) with steering vectors errors.

Fig. 12. ROC curves for the KASSPER data set.
Fig. 13. ROC curves for the KASSPER data set when using the IAA-based median detector and the AMF detector with $R_{CN}$.