CAPON BEAMFORMING IN THE PRESENCE OF STEERING VECTOR ERRORS AND COHERENT SIGNALS

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ABSTRACT

It is well-known that the standard Capon beamformer (SCB) suffers from severe signal cancellation when the knowledge of the signal-of-interest (SOI) steering vector is imprecise, the snapshot number is small (which can also be viewed as a steering vector error problem), or the interference is correlated with the SOI. Hence the SCB performs poorly in some applications, such as in a global positioning system (GPS), where SOI steering vector errors and coherent multipath interferences exist. In this paper, we propose a Capon beamformer that is robust against both SOI steering vector errors and coherent interferences provided that the directions of arrival (DOAs) of the coherent multipaths are approximately known relative to the DOA of SOI. Numerical examples are presented to demonstrate the effectiveness of the proposed coherent robust Capon beamformer, which we will designate by the acronym CRCB.

1. INTRODUCTION

The standard Capon beamformer (SCB) adaptively selects the weight vector to minimize the output power subject to the constraint that the signal of interest (SOI) does not suffer from any distortion. However, the SCB suffers from severe signal cancellation in the presence of coherent interferences or SOI steering vector errors.

To mitigate the signal cancellation problem caused by the SOI steering vector error, many robust methods have been proposed (see [1] and the references therein). However, most of these methods are rather *ad hoc* in that the choice of their parameters is not directly related to the uncertainty of the steering vector. Only recently have some methods with a clear theoretical background been proposed (see, for example, [2] [3] [4][5] and [6]). It has been proven in [6] that despite the apparent differences between the methods in [3] [4] and that in [5] [6], these robust Capon beamforming (RCB) approaches give the same weight vector, with the RCB in [6] being somewhat more computationally efficient. However, like SCB, the aforementioned RCB approaches perform poorly in the presence of coherent (or correlated) multipaths.

To mitigate the signal cancellation problem caused by coherent interferences, various methods have been proposed including spatial smoothing [7], SOI subtraction [8] and an ML-based method [9]. Among these methods, spatial smoothing reduces the array aperture and hence resolution; SOI subtraction requires the spatial knowledge of SOI and the ML-based method makes assumptions about the noise covariance and the number of sources and is rather sensitive to mismodeling. In addition, all these methods assume a uniform linear array (ULA) and no steering vector error.

In some applications, such as in a global positioning system (GPS) [10], the array is not necessary a ULA, SOI steering vector errors always exist (owing to a variety of factors such as steering angle errors and array calibration errors), and coherent interferences may also be present (owing to the multipath effect). In this paper, we propose a Capon beamformer that is robust against both SOI steering vector errors and coherent interferences. The main assumption underlying our coherent robust Capon beamformer (CRCB) is that the DOAs of the multipath interferences are known relative to the DOA of SOI (which is the case, e.g., in the GPS).

2. STANDARD AND ROBUST CAPON BEAMFORMING

Consider an array, comprising M sensors, whose output vector is described by the equation:

$$\mathbf{x}(n) = \mathbf{a}_0 s_0(n) + \sum_{k=1}^{K} \mathbf{a}_k s_k(n) + \mathbf{e}(n), \quad n = 1, 2, \cdots, N \tag{1}$$

where N denotes the number of snapshots, $s_0(n)$ is the SOI, $\mathbf{a}_0$ (with $\|\mathbf{a}_0\|^2 = M$) is the steering vector of the SOI, $\{s_k(n)\}_{k=1}^{K}$ are the K interferences, $\mathbf{a}_k$ denotes the steering

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vector of the kth interference, and $\mathbf{e}(n)$ is a noise vector with an unknown covariance matrix.

The covariance matrix of the received data vector is denoted by:

$$\mathbf{R} = E\left[\mathbf{x}(n)\mathbf{x}^*(n)\right] \tag{2}$$

where $E[\cdot]$ denotes the expectation and $(\cdot)^*$ stands for the conjugate transpose. In practical applications, $\mathbf{R}$ is replaced by the sample covariance matrix $\hat{\mathbf{R}}$, where

$$\hat{\mathbf{R}} = \frac{1}{N}\sum_{n=1}^{N}\mathbf{x}(n)\mathbf{x}^*(n) \tag{3}$$

The original formulation of the standard Capon beamformer (SCB) is as follows (see, e.g., [11]:

$$\min_{\mathbf{w}} \mathbf{w}^*\mathbf{R}\mathbf{w} \qquad \text{subject to} \qquad \mathbf{w}^*\mathbf{a}_0 = 1 \tag{4}$$

As is well known, the solution to (4) leads to the following estimate of the power of SOI:

$$\hat{\sigma}^2 = 1/\mathbf{a}_0^*\mathbf{R}^{-1}\mathbf{a}_0 \tag{5}$$

Interestingly, one equivalent formulation of the SCB problem, whose solution is also given by (5), is as follows (see [5][12]):

$$\max_{\sigma^2} \sigma^2 \quad \text{subject to} \quad \mathbf{R} - \sigma^2\mathbf{a}\mathbf{a}^* \geq \mathbf{0} \tag{6}$$

The aim of SCB is to pass the signal with steering vector $\mathbf{a}_0$ undistorted and, at the same time, minimize the output power. Assume that the interference $s_k(n)$ with steering vector $\mathbf{a}_k$ is coherent with the SOI; then $s_0(n)$ and $s_k(n)$ can be viewed as one signal $s_0(k)$ with steering vector $\mathbf{a}_0 + \alpha\mathbf{a}_k$, where $\alpha = s_k(n)/s_0(n)$. Since $\mathbf{a}_0 + \alpha\mathbf{a}_k$ is in general rather different from $\mathbf{a}_0$, the SOI will be treated as an interference and hence it will be suppressed. This observation also approximately holds for highly correlated signals. Hence SCB fails to function properly in the presence of highly correlated or coherent interferences. A similar argument can be used to explain why SCB does not function properly in the presence of steering vector errors either.

The recently proposed robust Capon beamformer (RCB) in [6] allows an imprecise knowledge of the steering vector. More specifically, the only knowledge assumed about $\mathbf{a}_0$ is that it belongs to the following uncertainty set:

$$\mathbf{a}_0 = \mathbf{B}\mathbf{u} + \bar{\mathbf{a}}, \quad \|\mathbf{u}\|^2 \leq \epsilon \tag{7}$$

where ϵ is a user parameter, $\mathbf{B}$ is an $M \times L$ matrix $(L \leq M)$ with full column rank, $\mathbf{u}$ is an arbitrary norm-constrained vector, and $\bar{\mathbf{a}}$ is the assumed steering vector. The set described by (7) is an M-dimensional ellipsoid when $L = M$ and a degenerated ("flat") ellipsoid when $L < M$. With (6) and (7) in mind, the RCB is formulated as a covariance matrix fitting problem [5][6] with an extra steering vector constraint as follows:

$$\max_{\mathbf{a},\sigma^2} \sigma^2 \quad \text{subject to} \qquad \mathbf{R} - \sigma^2\mathbf{a}\mathbf{a}^* \geq \mathbf{0}$$
$$\mathbf{a} = \mathbf{B}\mathbf{u} + \bar{\mathbf{a}}, \quad \|\mathbf{u}\|^2 \leq \epsilon \tag{8}$$

where σ^2 denotes the SOI power that we want to determine. The above problem can be simplified to (see, e.g., [5][6] and also the equations (12)-(15) below):

$$\min_{\mathbf{a}} \mathbf{a}^*\mathbf{R}^{-1}\mathbf{a} \quad \text{s.t.} \quad \mathbf{a} = \mathbf{B}\mathbf{u} + \bar{\mathbf{a}}, \quad \|\mathbf{u}\|^2 \leq \epsilon \tag{9}$$

The solution to (8) leads to a beamformer belonging to an extended class of diagonally loaded Capon methods, which has an excellent performance provided that no coherent interference exists [6]. However, like SCB, the RCB also fails to function properly in the presence of interferences which are highly correlated (or coherent) with the SOI.

3. COHERENT ROBUST CAPON BEAMFORMING

From now onward we will assume that some of the interferences in the array output equation, (1), are highly correlated (or coherent) with the SOI. The contribution of these interferences to the array output lies in the range space generated by their steering vectors. The basic assumption that underlines the derivation of our coherent robust Capon beamformer (CRCB) is that we have an approximate knowledge of the aforementioned range space. Specifically, we assume that we can determine a matrix $\mathbf{V}$ of size $M \times \bar{K}(\bar{K} < M)$, which is such that its range space is a good approximation of the range space generated by the interferences in (1) that are coherent or highly correlated with the SOI. Making use of $\mathbf{V}$ we can write the effective (or equivalent) steering vector of the SOI and its coherent multipaths as

$$\mathbf{a} + \mathbf{V}\mathbf{b} \tag{10}$$

where $\mathbf{a}$ is the (uncertain) steering vector of SOI, which is assumed to belong to the set in (7), and $\mathbf{b}$ is an unknown $\bar{K} \times 1$ vector whose elements equal the ratios between the multipath interferences and the SOI. Our motivating example for making the above assumption that led to (10) was a GPS application employing a vertical array, in which case if the SOI arrives from an angle equal to θ_0 then its coherent multipath tends to arrive approximately from $-\theta_0$ (see, e.g., [10] and the references therein). In such a case we can easily find a matrix $\mathbf{V}$ that satisfies the previously stated requirement.

Given the fact that in the present case the effective steering vector of SOI is (10), as explained above, we can modify the covariance fitting problem in (8) as follows:

$$\max_{\mathbf{a},\mathbf{b},\sigma^2} \sigma^2 \quad \text{s.t.} \qquad \mathbf{R} - \sigma^2(\mathbf{a}+\mathbf{V}\mathbf{b})(\mathbf{a}+\mathbf{V}\mathbf{b})^* \geq 0$$
$$\mathbf{a} = \mathbf{B}\mathbf{u} + \bar{\mathbf{a}}, \quad \|\mathbf{u}\|^2 \leq \epsilon \tag{11}$$

Here both ϵ, $\mathbf{B}$ and $\mathbf{V}$ must be chosen by the user according to the *a priori* knowledge that is available about the errors in the steering vector and the multipath interference. As we will illustrate later on via numerical examples, the choice of neither ϵ nor $\mathbf{B}$ nor $\mathbf{V}$ is a critical issue as long as these user parameters are chosen in a "reasonable" manner.

A derivation similar to that in [5][6] yields the following series of easily checked equivalences (hereafter, $\mathbf{R}^{-1/2}$ denotes the Hermitian square root of $\mathbf{R}^{-1}$):

$$\mathbf{R} - \sigma^2(\mathbf{a}+\mathbf{V}\mathbf{b})(\mathbf{a}+\mathbf{V}\mathbf{b})^* \geq 0 \Leftrightarrow \tag{12}$$

$$\mathbf{I} - \sigma^2\mathbf{R}^{-1/2}(\mathbf{a}+\mathbf{V}\mathbf{b})(\mathbf{a}+\mathbf{V}\mathbf{b})^*\mathbf{R}^{-1/2} \geq 0 \Leftrightarrow \tag{13}$$

$$1 - \sigma^2(\mathbf{a}+\mathbf{V}\mathbf{b})^*\mathbf{R}^{-1}(\mathbf{a}+\mathbf{V}\mathbf{b}) \geq 0 \Leftrightarrow \tag{14}$$

$$\sigma^2 \leq \frac{1}{(\mathbf{a}+\mathbf{V}\mathbf{b})^*\mathbf{R}^{-1}(\mathbf{a}+\mathbf{V}\mathbf{b})} = \hat{\sigma}^2 \tag{15}$$

Hence, for any fixed $\mathbf{a}$ and $\mathbf{b}$, the above $\hat{\sigma}^2$ is the solution to (11), which means that (11) can be reduced to the following problem

$$\begin{aligned} \min_{\mathbf{a},\mathbf{b}} \quad & (\mathbf{a}+\mathbf{V}\mathbf{b})^*\mathbf{R}^{-1}(\mathbf{a}+\mathbf{V}\mathbf{b}) \\ \text{s.t.} \quad & \mathbf{a} = \mathbf{B}\mathbf{u} + \bar{\mathbf{a}}, \quad \|\mathbf{u}\|^2 \leq \epsilon \end{aligned} \tag{16}$$

For any fixed $\mathbf{a}$, minimizing (16) w.r.t. $\mathbf{b}$ gives:

$$\hat{\mathbf{b}} = -(\mathbf{V}^*\mathbf{R}^{-1}\mathbf{V})^{-1}\mathbf{V}^*\mathbf{R}^{-1}\mathbf{a} \tag{17}$$

Inserting (17) into (16), we obtain the following problem with $\mathbf{a}$ as the sole variable:

$$\begin{aligned} \min_{\mathbf{a}} \quad & \mathbf{a}^*\mathbf{\Gamma}\mathbf{a} \\ \text{s.t.} \quad & \mathbf{a} = \mathbf{B}\mathbf{u} + \bar{\mathbf{a}}, \quad \|\mathbf{u}\|^2 \leq \epsilon \end{aligned} \tag{18}$$

where

$$\begin{aligned} \mathbf{\Gamma} &= \mathbf{R}^{-1} - \mathbf{R}^{-1}\mathbf{V}(\mathbf{V}^*\mathbf{R}^{-1}\mathbf{V})^{-1}\mathbf{V}^*\mathbf{R}^{-1} \\ &\triangleq \mathbf{R}^{-1/2}\mathbf{P}^{\perp}_{\mathbf{R}^{-1/2}\mathbf{V}}\mathbf{R}^{-1/2} \end{aligned} \tag{19}$$

Note that $\mathbf{P}^{\perp}_{\mathbf{R}^{-1/2}\mathbf{V}}$ above is the orthogonal projector onto the null space of $(\mathbf{R}^{-1/2}\mathbf{V})^*$.

The following calculation provides some additional insights into (18). Let $\mathbf{G}$ be a basis of the null space of $\mathbf{V}^*$; then $\mathbf{R}^{1/2}\mathbf{G}$ is a basis for the null space of $(\mathbf{R}^{-1/2}\mathbf{V})^*$. Consequently we have $\mathbf{P}^{\perp}_{\mathbf{R}^{-1/2}\mathbf{V}} = \mathbf{R}^{1/2}\mathbf{G}(\mathbf{G}^*\mathbf{R}\mathbf{G})^{-1}\mathbf{G}^*\mathbf{R}^{1/2}$. Hence (18) can be rewritten as

$$\begin{aligned} \min_{\mathbf{a}} \quad & \mathbf{a}^*\mathbf{G}(\mathbf{G}^*\mathbf{R}\mathbf{G})^{-1}\mathbf{G}^*\mathbf{a} \\ \text{s.t.} \quad & \mathbf{a} = \mathbf{B}\mathbf{u} + \bar{\mathbf{a}}, \quad \|\mathbf{u}\|^2 \leq \epsilon \end{aligned} \tag{20}$$

If we "pre-filter" the received data defined in (1) in the spatial domain via $\mathbf{G}^*$, we get

$$\begin{aligned} \mathbf{y}(n) &= \mathbf{G}^*\mathbf{x}(n) \\ &= \mathbf{G}^*\mathbf{a}_0 s_0(n) + \sum_{k=1}^{K} \mathbf{G}^*\mathbf{a}_k s_k(n) + \mathbf{G}^*\mathbf{e}(n), \\ & \quad n = 1, 2, \cdots, N \end{aligned} \tag{21}$$

Note that (20) is equivalent to applying the RCB method in [6] (see (9)) to the pre-filtered data model (21) in which the multipath interferences were filtered out (with the only difference that the constraint in (20) is imposed on $\mathbf{a}$, and not on $\mathbf{G}^*\mathbf{a}$ as it would be in the case of (21)). Since (20) is more intuitive, we will concentrate on this formulation in the sequel. We can solve (20) using the Lagrange multiplier methodology similar to what has been done in [6] to solve the look-alike problem in (9) (we refer the reader to [6] for details).

Let $\hat{\mathbf{a}}_0$ denote the solution to (20). As already stated, we assume that $\|\mathbf{a}_0\|^2 = M$. As in general $\|\hat{\mathbf{a}}_0\|^2 \neq M$, we can use the following scaled version of $\hat{\mathbf{a}}_0$ in lieu of $\hat{\mathbf{a}}_0$:

$$\mathbf{a} = \hat{\mathbf{a}}_0 M^{1/2}/\|\hat{\mathbf{a}}_0\| \tag{22}$$

Inserting (22) in the power estimate expression derived previously, viz. $\hat{\sigma}^2 = 1/\mathbf{a}^*\mathbf{G}(\mathbf{G}^*\mathbf{R}\mathbf{G})^{-1}\mathbf{G}^*\mathbf{a}$, we obtain

$$\hat{\sigma}_0^2 = \frac{\|\hat{\mathbf{a}}_0\|^2}{M\hat{\mathbf{a}}_0^*\mathbf{G}(\mathbf{G}^*\mathbf{R}\mathbf{G})^{-1}\mathbf{G}^*\hat{\mathbf{a}}_0} \tag{23}$$

To estimate the signal waveform, we can use the weight vector (for the data $\{\mathbf{x}(n)\}$):

$$\hat{\mathbf{w}}_0 = \mathbf{G}(\mathbf{G}^*\mathbf{R}\mathbf{G})^{-1}\mathbf{G}^*\hat{\mathbf{a}}_0 \tag{24}$$

In the previous expression for $\hat{\mathbf{w}}_0$, we omitted a scaling factor, which is possible because the signal-to-interference-plus-noise ratio (SINR) is insensitive to the scaling of $\hat{\mathbf{w}}_0$.

4. DISCUSSION

4.1. Performance Aspects

As we have pointed out, the CRCB uses a spatial filter which approximately nulls the coherent interferences before applying the RCB of [5][6]. The cost is a loss of $\bar{K}$ degrees of freedom of the beamformer (observe that $\hat{\mathbf{w}}_0$ in (24) lies in an $(M - \bar{K})$ dimensional subspace). Hence it is desirable to determine a matrix $\mathbf{V}$ with the least possible number of columns, $\bar{K}$.

To simplify the following discussion (as well as the numerical examples in the next section), let us assume that the only multipath interference in (1) is $s_1(n)$. We see from (21) that if the steering vector of the coherent interference is not nulled out completely by $\mathbf{G}$, i.e., $\mathbf{G}^*\mathbf{a}_1 \neq 0$, the residual steering vector $\mathbf{G}^*\mathbf{a}_1$ is combined with the SOI steering vector $\mathbf{G}^*\mathbf{a}_0$. In this case, the effect of the residual coherent multipath amounts to adding some uncertainty to the steering vector of the SOI. We will show in the numerical examples that this added uncertainty is not a problem for the proposed CRCB since its RCB part is designed to handle steering vector errors. Note that in the present case we can simply choose $\mathbf{V} = \bar{\mathbf{a}}_1$, where $\bar{\mathbf{a}}_1$ is the assumed steering vector of the coherent multipath, which only causes the loss of one degree of freedom. (See the numerical examples.)

4.2. Computational Aspects

The CRCB is computationally efficient if steered at a single angle, such as when we want to estimate the power coming from a given direction. However CRCB may be computationally intensive for calculating the spatial spectrum since either (18) or (20) requires the computation of matrix inverses for every considered angle. In the following, we propose a fast direction-of-arrival (DOA) estimation method for coherent signals based on a covariance matrix fitting approach similar to that used to formulate (11). Considering the GPS scenario mentioned above, in which for a SOI at DOA= θ there exists a multipath interference at DOA= $-\theta$, let

$$\mathbf{A}(\theta) = [\mathbf{a}(\theta), \mathbf{a}(-\theta)] \tag{25}$$

Then we want to solve the problem (cf. (11))

$$\max_{\alpha,\sigma^2} \sigma^2 \quad \text{s.t.} \quad \mathbf{R} - \sigma^2 \mathbf{A}(\theta)[1\ \alpha]^T[1\ \alpha^*]\mathbf{A}^*(\theta) \geq 0. \tag{26}$$

According to (15), we have

$$\hat{\sigma}^2 = \frac{1}{[1\ \alpha^*]\mathbf{A}^*(\theta)\mathbf{R}^{-1}\mathbf{A}(\theta)[1\ \alpha]^T} \tag{27}$$

Hence the problem left to solve is

$$\min_{\alpha} [1\ \alpha^*]\mathbf{A}^*(\theta)\mathbf{R}^{-1}\mathbf{A}(\theta)[1\ \alpha]^T \tag{28}$$

Minimizing (28) w.r.t. α gives the following estimate of the spatial spectrum:

$$\hat{\sigma}^2(\theta) = \frac{h_{22}(\theta)}{h_{11}(\theta)h_{22}(\theta) - |h_{12}(\theta)|^2} \tag{29}$$

where $h_{ij}(\theta)$ is the $(i,j)^{\text{th}}$ element of the 2x2 matrix

$$\mathbf{H}(\theta) = \mathbf{A}^*(\theta)\mathbf{R}^{-1}\mathbf{A}(\theta) \tag{30}$$

Note that this method assumes no steering vector error. It simply relies on the fact that the DOA estimates obtained via the SCB approach are not as sensitive to steering vector errors as the signal power estimates (see e.g. [6]). The matrix inverse $\mathbf{R}^{-1}$ in (28) needs to be calculated only once for all DOAs. In the sequel, we will refer to this modified standard Capon beamformer as MSCB. Numerical examples (see the next section) suggest that MSCB can well estimate the DOAs of the incident signals in the presence of both coherent interferences and steering vector errors. Hence in practice we can use the MSCB to estimate the DOA of the SOI, and then estimate the SOI power and waveform using the CRCB method at the DOA estimate provided by the MSCB.

5. NUMERICAL EXAMPLES

In this section, we will present several numerical examples for a GPS scenario in which a SOI at angle θ_0 has a possible coherent multipath at $-\theta_0$. In all the examples considered below, we assume a uniform linear array comprising $M = 10$ sensors with half-wavelength sensor spacing and a number of snapshots $N = 40$. (We consider a ULA here just for simplicity. The proposed CRCB method can be applied to an arbitrary array as can be seen from the derivations in Section 3.) The SOI is generated as a constant signal, i.e., $s_0(n) = 1$. The interferences which are uncorrelated with the SOI and the noise are generated as temporally white Gaussian processes. In addition, the noise is also assumed to be spatially white. Also we use $\mathbf{B} = \mathbf{I}$ in (11), assuming that there is no *a priori* information that would allow us to make a better choice.

Figure 1 illustrates the effectiveness of the MSCB as a DOA estimation method. In this case an interference uncorrelated with the SOI comes from $-45°$ with power 40 dB, the SOI comes from $-30°$ with power 30 dB, and a coherent multipath impinges from $29°$ with power 27 dB. The assumed array manifold vector $\bar{\mathbf{a}}_k$, for $k = 0, 1, \ldots, K$, is realized by adding an independent random vector to the true (unknown) array manifold vector $\mathbf{a}_k$ such that $\|\mathbf{a}_k - \bar{\mathbf{a}}_k\|^2 = 0.2$. The MSCB method provides fairly accurate DOA estimates, unlike the SCB which fails to detect the SOI and its multipath. A data independent beamformer using a Dolph-Chebyshev window with a peak sidelobe level of -30 dB does not have the necessary resolution to separate the two signals from $-45°$ and $-30°$. The false peaks around $0°$ and $\pm 90°$ in the MSCB spectrum are due to the rank deficiency of the matrix $\mathbf{A}(\theta)$ at these angles.

We next consider a scenario where an interference uncorrelated with the SOI comes from $-40°$ with power 40 dB, the SOI comes from $-10°$ with power 30 dB, and a coherent interference impinges from $9°$ with power 27 dB. We assume $\epsilon = 0.3$ while in actuality we have $\|\mathbf{a}_k - \bar{\mathbf{a}}_k\|^2 = 0.1$, for $k = 0, 1, \ldots, K$. Figure 2 confirms the superior performance of the CRCB in the presence of coherent interferences, especially as a power estimation method. Note that, like for MSCB, the spurious peaks in the CRCB spatial spectrum at $\theta = 0°$ and $\theta = \pm 90°$ are due to the fact that the SOI steering vector, $\mathbf{a}(\theta)$, and the steering vector of the coherent interference, $\mathbf{a}(-\theta)$, are identical at the aforementioned angles. When this happens, the vector $\mathbf{G}^*\mathbf{a}$ in the denominator of the CRCB power estimate (see (23)) takes on rather small values and hence $\hat{\sigma}^2$ becomes rather large.

In Figures 3 and 4, we show how the SINR varies w.r.t. the assumed squared norm of the steering vector error, ϵ, when the true norm of this error is $\|\mathbf{a}_k - \bar{\mathbf{a}}_k\|^2 = 1.0$, for $k =$

$0, 1, \ldots, K$. The SINR is given by

$$\text{SINR} = \frac{\sigma_0^2 |\hat{\mathbf{w}}_0^* \mathbf{a}_{\text{eq}}|^2}{\hat{\mathbf{w}}_0^* \mathbf{R}_n \hat{\mathbf{w}}_0} \quad (31)$$

where

$$\mathbf{a}_{\text{eq}} = \mathbf{a}_0 + \alpha \mathbf{a}_1 \quad (32)$$

is the equivalent steering vector for the SOI and its coherent multipath interference. We calculate the SINR by taking the average over 100 Monte Carlo trials. For Figure 3 the incident signals are from $-40°$ with power 40 dB, $-30°$ with power 30 dB, and $29°$ with power 27 dB. The signals from $29°$ and $-30°$ are coherent. For Figure 4 the incident signals are from $-40°$ with power 40 dB, $-20°$ with power 30 dB, and $19°$ with power 27 dB. The signals from $19°$ and $-20°$ are coherent. In both figures, the solid line with circles corresponds to the ideal case in which $\mathbf{a}_1$ is known precisely and $\mathbf{V} = \mathbf{a}_1$. Thus the coherent interference is nulled out perfectly in this case. The dashed line with asterisks is for the case of $\mathbf{V} = [\bar{\mathbf{a}}(-\theta - 0.5°), \bar{\mathbf{a}}(-\theta + 0.5°)]$, whereas the solid line with triangles corresponds to $\mathbf{V} = \bar{\mathbf{a}}(-\theta)$ (where θ denotes the DOA of SOI; $\theta = -30°$ for Figure 3 and $\theta = -20°$ for Figure 4). Both figures show that the approximation errors in the choice of the matrix $\mathbf{V}$ can be compensated for by increasing the parameter ϵ. Note that simply choosing $\mathbf{V} = [\bar{\mathbf{a}}(-\theta)]$ is good enough for these examples. Also note that the SINR is not very sensitive to the choice of ϵ. It is interesting to observe that the SINR decreases more in Figure 3 than in Figure 4 for large ϵ. This is so because in the scenario of Figure 3, an interference is closer to the desired signal and increasing ϵ reduces the capability of CRCB to suppress this nearby interference.

6. CONCLUSIONS

We have proposed a coherent robust Capon beamformer (CRCB) which works well in the presence of both SOI steering vector errors and coherent interferences provided that the steering vectors of the interferences are approximately known relative to the SOI steering vector. Essentially, the CRCB exploits the *a priori* information on the coherent interferences to approximately null them before applying the RCB. The CRCB has a computational burden similar to that of the SCB when used for a single DOA (the beamforming mode); however, CRCB may be computationally intensive when used for many DOA's (the DOA estimation mode). To reduce the computational complexity in the later case, we have proposed a modified standard Capon beamformer (MSCB) which can be used to estimate the DOA's quite accurately in the presence of both coherent multipaths and steering vector errors. Using MSCB and CRCB together, we can locate the source signals of interest (via MSCB) and then estimate their powers and waveforms (via CRCB), in a computationally efficient manner.

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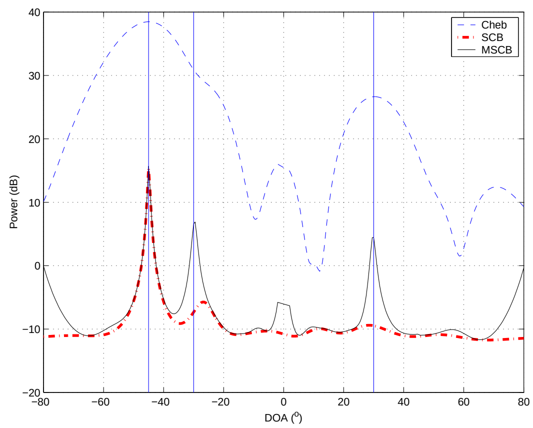

Figure 1: Spatial spectrum estimates when $\epsilon = 0.2$. The incident signals come from $-45°$ with power 40 dB, $-30°$ with power 30 dB, and $29°$ with power 27 dB. The signals from $29°$ and $-30°$ are coherent.

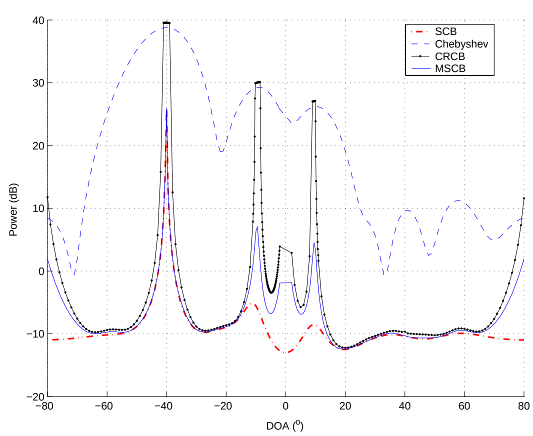

Figure 2: Spatial spectrum estimates when the array calibration errors are $\|\mathbf{a}_k - \bar{\mathbf{a}}_k\|^2 = 0.1$ for all k and the assumed ϵ is 0.3. The incident signals come from $-40°$ with power 40 dB, $-10°$ with power 30 dB, and $9°$ with power 27 dB. The signals from $9°$ and $-10°$ are coherent.

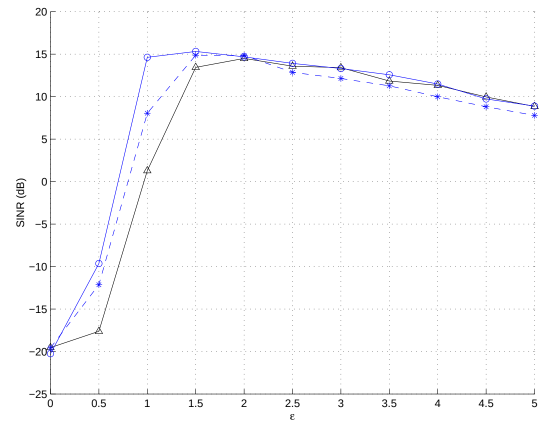

Figure 3: SINR vs. ϵ. The array calibration errors are $\|\mathbf{a}_k - \bar{\mathbf{a}}_k\|^2 = 1.0$ for all k. The incident signals come from $-40°$ with power 40 dB, $-30°$ with power 30 dB, and $29°$ with power 27 dB. The signals from $29°$ and $-30°$ are coherent. The meaning of the three curves is explained in the text.

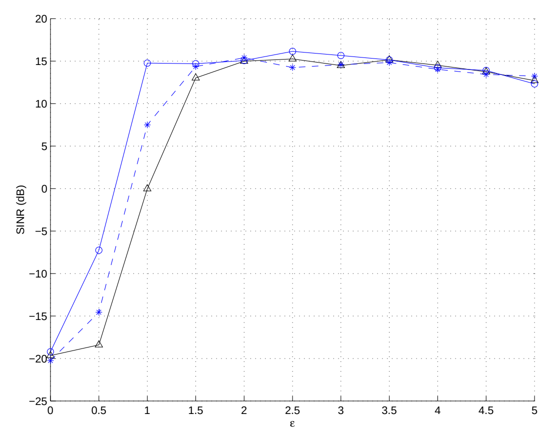

Figure 4: SINR vs. ϵ. The array calibration errors are $\|\mathbf{a}_k - \bar{\mathbf{a}}_k\|^2 = 1.0$ for all k. The incident signals are from $-40°$ with power 40 dB, $-20°$ with power 30 dB, and $19°$ with power 27 dB. The signals from $-20°$ and $19°$ are coherent. The meaning of the three curves is explained in the text.