ADAPTABLE CHANNEL DECOMPOSITION FOR MIMO COMMUNICATIONS

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ABSTRACT

Assuming the availability of the channel state information at the transmitter (CSIT) and receiver (CSIR), we consider the joint optimal transceiver design for multi-input multi-output (MIMO) communication systems. Using our recently developed generalized triangular decomposition (GTD), we propose a scheme, which we refer to as adaptable channel decomposition (ACD), to decompose a MIMO channel into multiple subchannels with prescribed capacities, or equivalently, signal-to-interference-and-noise ratios (SINR). We also determine the subchannel capacity region such that the channel decomposition is capacity lossless. This scheme is particularly relevant to the applications where independent data streams with different quality-of-services (QoS) share the same MIMO channel.

1. INTRODUCTION

For a multi-input multi-output (MIMO) communication system, if the communication environment is relatively stationary, such as for the indoor wireless local area networks (WLAN) and the bonded digital subscriber lines (DSL) communications, the channel state information at transmitter (CSIT) is possible via feedback or the reciprocal principle when time division duplex (TDD) is used. Assuming the availability of both CSIT and the channel state information at receiver (CSIR), considerable research has been devoted to the optimal MIMO transceiver design (see [1][2] and the reference therein). Two classes of MIMO transceiver designs have been proposed, including linear transceiver designs and nonlinear schemes. While the linear transceivers have attracted much attention during the past several years, the more recent nonlinear schemes, which are based on either a VBLAST detector or a dirty paper (DP) precoder, can significantly outperform their linear counterparts in terms of both BER performance and channel throughput [2]. All these aforementioned MIMO transceiver designs focus on improving the communication quality subject to power constraints. In this paper, we tackle a more information-theoretic aspect of the MIMO transceiver design problem. We regard a MIMO transceiver design as a way of decomposing a MIMO channel into multiple subchannels. Using our recently developed generalized triangular decomposition (GTD), we propose a scheme, which we refer to as adaptable channel decomposition (ACD), to decompose a MIMO channel into multiple subchannels with prescribed capacities, or equivalently, signal-to-interference-and-noise ratios (SINR). We also determine the subchannel capacity region such that the channel decomposition is capacity lossless. This scheme is particularly relevant to the applications where independent data streams with different quality-of-services (QoS) share the same MIMO channel [3]. The ACD scheme has two implementation forms. One is the combination of a linear precoder and a minimum mean-squared-error VBLAST (MMSE-VBLAST) detector, which is referred to as ACD-VBLAST, and the other includes a DP precoder and a linear equalizer followed by a DP decoder, which we refer to as ACD-DP.

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2. CHANNEL MODEL AND PRELIMINARIES

2.1. Channel Model

We consider a MIMO communication system with M_t transmitting and M_r receiving antennas in a frequency flat fading channel. The sampled baseband signal is given by

$$\mathbf{y} = \mathbf{H}\mathbf{F}\mathbf{x} + \mathbf{z}, \tag{1}$$

where $\mathbf{x} \in \mathbb{C}^{L\times 1}$ is the information symbol vector precoded by the linear precoder. $\mathbf{F} \in \mathbb{C}^{M_t\times L}$ and $\mathbf{y} \in \mathbb{C}^{M_r\times 1}$ is the received signal and $\mathbf{H} \in \mathbb{C}^{M_r\times M_t}$ is the channel matrix with rank K. [1] We assume $E[\mathbf{x}\mathbf{x}^*] = \sigma_x^2\mathbf{I}_L$, where $E[\cdot]$ is the expected value and $\mathbf{I}_L$ denotes the identity matrix with dimension L and $\mathbf{z} \sim N(0, \sigma_z^2\mathbf{I}_{M_r})$ is the circularly symmetric complex Gaussian noise. We define the input SNR as

$$\rho = \frac{E[\mathbf{x}^*\mathbf{F}^*\mathbf{F}\mathbf{x}]}{\sigma_z^2} = \frac{\sigma_x^2}{\sigma_z^2}\mathrm{Tr}\{\mathbf{F}^*\mathbf{F}\} \triangleq \frac{1}{\alpha}\mathrm{Tr}\{\mathbf{F}^*\mathbf{F}\}, \tag{2}$$

[1] Throughout this paper, we assume $L \geq K$ and can be arbitrarily large. If $L < K$, the precoder will suffer from capacity loss in general.

where $(\cdot)^*$ is the conjugate transpose, and $\text{Tr}\{\cdot\}$ stands for the trace of a matrix. Throughout this paper, we assume perfect CSIR and CSIT, i.e., $\mathbf{H}$ is known exactly at both the transmitter and receiver.

2.2. Majorization

Definition 1. *For $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, if*

$$\sum_{i=1}^{j} x_{[i]} \leq \sum_{i=1}^{j} y_{[i]}, \quad 1 \leq j \leq n \tag{3}$$

with equality hold for $j = n$, where the subscript $[i]$ denotes the ith largest element of the sequence, we say that $\mathbf{x}$ is majorized by $\mathbf{y}$ and denote $\mathbf{x} \prec \mathbf{y}$, or equivalently, $\mathbf{y} \succ \mathbf{x}$.

Definition 2. *For $\mathbf{x}, \mathbf{y} \in \mathbb{R}_+^n$, if*

$$\prod_{i=1}^{j} x_{[i]} \leq \prod_{i=1}^{j} y_{[i]}, \quad 1 \leq j \leq n \tag{4}$$

with equality hold for $j = n$, we say that $\mathbf{x}$ is multiplicatively *majorized by $\mathbf{y}$ and denote $\mathbf{x} \prec_\times \mathbf{y}$, or equivalently, $\mathbf{y} \succ_\times \mathbf{x}$.*

Obviously, if $\mathbf{x} \prec_\times \mathbf{y}$, then $\log \mathbf{x} \prec \log \mathbf{y}$.

Theorem 2.1 (GTD theorem). *Let $\mathbf{H} \in \mathbb{C}^{m \times n}$ have rank K with singular values $\lambda_{H,1} \geq \lambda_{H,2} \geq \ldots \geq \lambda_{H,K} > 0$. There exists an upper triangular matrix $\mathbf{R} \in \mathbb{C}^{K \times K}$ and matrices $\mathbf{Q}$ and $\mathbf{P}$ with orthonormal columns such that $\mathbf{H} = \mathbf{QRP}^*$ if and only if the diagonal elements of $\mathbf{R}$ satisfy*

$$\{|r_{ii}|\}_{i=1}^K \prec_\times \{\lambda_{H,i}\}_{i=1}^K \tag{5}$$

Proof. See [4]. □

In [4], we propose a computationally efficient and numerically stable algorithm to achieve the GTD predicted by Theorem 2.1.

2.3. Closed-Form Representation of MMSE-VBLAST

The derivation of the ACD scheme relies on the closed-form representation of the MMSE-VBLAST detector introduced in [5].

Consider the augmented matrix

$$\mathbf{H}_a = \begin{bmatrix} \mathbf{H} \\ \sqrt{\alpha}\mathbf{I}_{M_t} \end{bmatrix}_{(M_r+M_t)\times M_t} \tag{6}$$

with its QR decomposition

$$\mathbf{H}_a = \mathbf{Q}_{H_a}\mathbf{R}_{H_a} \triangleq \begin{bmatrix} \mathbf{Q}_{H_a}^u \\ \mathbf{Q}_{H_a}^l \end{bmatrix} \mathbf{R}_{H_a} \tag{7}$$

where $\mathbf{R}_{H_a} \in \mathbb{C}^{M_t \times M_t}$ is an upper triangular matrix with positive diagonal and $\mathbf{Q}_{H_a}^u \in \mathbb{C}^{M_r \times M_t}$. We can obtain the nulling vectors $\{\mathbf{w}_i^{\text{MMSE}}\}_{i=1}^K$ using $\mathbf{Q}_{H_a}^u$ and $\mathbf{R}_{H_a}$ as shown in the following lemma [5].

Lemma 2.1. *Let $\{\mathbf{q}_i\}_{i=1}^{M_t}$ denote the columns of $\mathbf{Q}_{H_a}^u$ and $\{r_{ii}\}_{i=1}^{M_t}$ the diagonal elements of $\mathbf{R}_{H_a}$ where $\mathbf{Q}_{H_a}^u$ and $\mathbf{R}_{H_a}$ are given in (7). The MMSE nulling vectors are*

$$\mathbf{w}_i^{\text{MMSE}} = r_{ii}^{-1}\mathbf{q}_i, \quad i = 1, 2, \ldots, M_t. \tag{8}$$

It is easy to verify that the output signal-to-interference-and-noise ratio (SINR) of the ith layer/subchannel (i.e., the signal corresponding to $\mathbf{h}_i$) obtained via the MMSE-VBLAST detector is $\rho_i = \mathbf{h}_i^*\mathbf{C}_i^{-1}\mathbf{h}_i$ where $\mathbf{C}_i = \sum_{j=1}^{i-1}\mathbf{h}_j\mathbf{h}_j^* + \alpha\mathbf{I}$.

The following lemma can be readily verified:

Lemma 2.2. *The diagonal of $\mathbf{R}_{H_a}$ given in (7) and $\{\rho_i\}_{i=1}^{M_t}$ satisfy*

$$\alpha(1 + \rho_i) = r_{ii}^2, \quad i = 1, 2, \ldots, M_t. \tag{9}$$

3. ADAPTABLE CHANNEL DECOMPOSITION

3.1. ACD-VBLAST

Now we are ready to establish the ACD scheme. Note that α defined in (2) can always be absorbed into $\mathbf{F}$. Hence without loss of generality, we let $\alpha = 1$ in the sequel. Let $\mathbf{H} = \mathbf{U\Lambda V}$ be the SVD. Consider the precoder $\mathbf{F}$

$$\mathbf{F} = \mathbf{V}\mathbf{\Phi}^{1/2}\mathbf{\Omega}^* \tag{10}$$

where $\mathbf{\Omega} \in \mathbb{C}^{L \times K}$ with $L \geq K$ and $\mathbf{\Omega}^*\mathbf{\Omega} = \mathbf{I}$ and $\mathbf{\Phi}$ is a diagonal matrix with its diagonal entries determined via "water filling" to be

$$\phi_k(\mu) = \left(\mu - \frac{1}{\lambda_{H,k}^2}\right)^+, \tag{11}$$

with μ being chosen such that $\sum_{k=1}^{K}\phi_k(\mu) = \rho\sigma_z^2/\sigma_x^2 = \rho$ and $(a)^+ = \max\{0, a\}$. Then the precoder $\mathbf{F}$ maximizes the overall channel throughput. The role of $\mathbf{\Omega}$ is demonstrated in the following theorem.

Theorem 3.1 (ACD Theorem). *Consider a MIMO channel of (1) with $\mathbf{F}$ given in (10). Suppose $L \geq K$. Let $\mathbf{c} \in \mathbb{R}^L$ be a zero vector with its first K elements replaced by $\{C_k\}_{k=1}^K$. Here $C_k = \log\left(1 + \lambda_{H,k}^2\phi_k\right)$. For all $\{R_k\}_{k=1}^L \prec \mathbf{c}$, there is a semi-unitary matrix $\mathbf{\Omega} \in \mathbb{C}^{L \times K}$ such that the combination of the linear precoder $\mathbf{F}$ and the MMSE-VBLAST detector yields L subchannels with capacities equal to $\{R_k\}_{k=1}^L$. Conversely, for any precoder $\mathbf{F}$ given in (10), the capacities of the subchannels obtained via MMSE-VBLAST, $\{R_k\}_{k=1}^L$, are majorized by $\mathbf{c}$.*

Proof. Given the precoder of (10), the virtual channel is

$$\mathbf{G} \triangleq \mathbf{HF} = \mathbf{U\Lambda}\Phi^{1/2}\mathbf{\Omega}^* \triangleq \mathbf{U\Sigma\Omega}^* \tag{12}$$

where $\mathbf{\Sigma} = \mathbf{\Lambda}\Phi^{1/2}$ is a diagonal matrix with diagonal elements

$$\sigma_i = \lambda_{H,i}{\phi_i}^{1/2} \quad i = 1, \ldots, K. \tag{13}$$

Then we obtain the augmented matrix

$$\mathbf{G}_a = \begin{bmatrix} \mathbf{U}\boldsymbol{\Sigma}\boldsymbol{\Omega}^* \\ \mathbf{I}_L \end{bmatrix} \tag{14}$$

which can be rewritten as

$$\mathbf{G}_a = \begin{bmatrix} \mathbf{I}_{M_r} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Omega}_0 \end{bmatrix} \begin{bmatrix} \mathbf{U}[\boldsymbol{\Sigma} \vdots \mathbf{0}_{K\times(L-K)}] \\ \mathbf{I}_L \end{bmatrix} \boldsymbol{\Omega}_0^*, \tag{15}$$

where $\boldsymbol{\Omega}_0 \in \mathbb{C}^{L\times L}$ is unitary with its first K columns forming $\boldsymbol{\Omega}$. Let $\mathbf{J}$ denote

$$\mathbf{J} \triangleq \begin{bmatrix} \mathbf{U}[\boldsymbol{\Sigma} \vdots \mathbf{0}_{K\times(L-K)}] \\ \mathbf{I}_L \end{bmatrix}. \tag{16}$$

Then the singular values of $\mathbf{J}$ are $\lambda_{J,i} = \sqrt{1+\sigma_i^2}, \quad i = 1,\ldots,K$, and 1 otherwise. According to Theorem 2.1, we can apply GTD to obtain

$$\mathbf{J} = \mathbf{Q}_J \mathbf{R}_J \mathbf{P}_J^* \tag{17}$$

if and only if the diagonal elements of $\mathbf{R}_J \in \mathbb{R}_+^{L\times L}$, which we denote as $\{r_{J,ii}\}_{i=1}^L$, satisfy

$$\{r_{J,ii}\}_{i=1}^L \prec_\times \{\lambda_{J,i}\}_{i=1}^L. \tag{18}$$

Inserting (17) into (15) yields

$$\mathbf{G}_a = \begin{bmatrix} \mathbf{I}_{M_r} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Omega}_0 \end{bmatrix} \mathbf{Q}_J \mathbf{R}_J \mathbf{P}_J^* \boldsymbol{\Omega}_0^*. \tag{19}$$

Let $\boldsymbol{\Omega}_0 = \mathbf{P}_J^*$ and

$$\mathbf{Q}_{G_a} = \begin{bmatrix} \mathbf{I}_{M_r} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Omega}_0^* \end{bmatrix} \mathbf{Q}_J. \tag{20}$$

Then (19) can be rewritten to be $\mathbf{G}_a = \mathbf{Q}_{G_a}\mathbf{R}_J$ which is the QR decomposition of $\mathbf{G}_a$. The semi-unitary matrix $\boldsymbol{\Omega}$ associated with $\mathbf{G}_a$ consists of the first K columns of $\boldsymbol{\Omega}_0$ (or $\mathbf{P}_J^*$). Next, we see from Lemma 2.2 and (18) that

$$\{1+\rho_i\}_{i=1}^L = \{r_{J,ii}^2\}_{i=1}^L \prec_\times \{\lambda_{J,i}^2\}_{i=1}^L \tag{21}$$

where $\lambda_{J,i}^2 = 1 + \lambda_{H,i}^2\phi_i$ for $i = 1,\ldots,K$ and 1 otherwise. Hence

$$\{R_i\}_{i=1}^L = \{\log(1+\rho_i)\}_{i=1}^L \prec \{\log(\lambda_{J,i}^2)\}_{i=1}^L = \mathbf{c}. \tag{22}$$

□

Theorem 3.1 gives the subchannel capacity region, which is $\{R_i\}_{i=1}^L \prec \mathbf{c}$. Moreover, it implies that ACD is capacity lossless within this region since $\sum_{i=1}^K C_i = \sum_{i=1}^L R_i$. The proof of Theorem 3.1 is insightful. Indeed, given the SVD of $\mathbf{H}$ and the "water filling" level $\boldsymbol{\Phi}^{1/2}$, we only need to calculate the GTD given in (17). Then we immediately obtain the linear precoder $\mathbf{F} = \mathbf{V}\boldsymbol{\Phi}^{1/2}\boldsymbol{\Omega}^*$, where $\boldsymbol{\Omega}$ consists of the first K columns of $\mathbf{P}_J^*$. Let $\mathbf{Q}_{G_a}^u$ denote the first M_r rows of $\mathbf{Q}_{G_a}$, or equivalently the first M_r rows of $\mathbf{Q}_J$ (cf. (20)). According to Lemma 2.1, the nulling vectors are calculated as $\mathbf{w}_i^{\text{MMSE}} = r_{J,ii}^{-1}\mathbf{q}_{G_a,i}, \quad i = 1,2,\ldots,L$, where $\mathbf{q}_{G_a,i}$ is the ith column of $\mathbf{Q}_{G_a}^u$.

3.2. ACD-DP

As a dual form of ACD-VBLAST, the ACD scheme can be implemented by using a DP precoder, which we refer to as ACD-DP. We note that for a system with high dimensionality, UCD-DP is a better choice than UCD-VBLAST since it is free of propagation error (see, for example, [6] for details).

4. ACD FOR MIMO COMMUNICATIONS WITH QOS CONSTRAINTS

Suppose we want to transmit $L \geq K$ independent data streams through a MIMO channel. Instead of multiplexing all the substreams in the time division manner to share the entire MIMO channel, we apply ACD to decompose a MIMO channel into multiple subchannels with their SNRs meeting the QoS requirements of the substreams and dedicate each subchannel to one substream. Meanwhile we wish to minimize the overall input power. We need to solve the following optimization problem.

$$\begin{array}{ll} \min_{\mathbf{F}} & \operatorname{tr}(\mathbf{F}\mathbf{F}^*) \\ \text{subject to} & \mathbf{G}_a \triangleq \begin{pmatrix} \mathbf{HF} \\ \mathbf{I} \end{pmatrix} = \mathbf{Q}_{G_a}\mathbf{R}_{G_a} \\ & \mathbf{r}_{G_a} = \{\sqrt{1+\rho_i}\}_{i=1}^L \end{array} \tag{23}$$

where $\mathbf{r}_{G_a} \in \mathbb{R}^L$ consists of the diagonal elements of $\mathbf{R}_{G_a}$. Without loss of generality, we assume here $\rho_1 \geq \rho_2 \geq \ldots \geq \rho_L$. From Theorem 3.1, we see that (23) can be reformulated as

$$\begin{array}{ll} \min_{\mathbf{F}} & \operatorname{tr}(\mathbf{F}\mathbf{F}^*) \\ \text{subject to} & \boldsymbol{\lambda}_{G_a} \succ_\times \{\sqrt{1+\rho_i}\}_{i=1}^L \\ & \mathbf{G}_a = \begin{pmatrix} \mathbf{HF} \\ \mathbf{I} \end{pmatrix} \end{array} \tag{24}$$

where $\boldsymbol{\lambda}_A$ stands for the singular values of $\mathbf{A}$. Given the SVDs $\mathbf{H} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{V}^*$ and $\mathbf{F} = \mathbf{V}_F\boldsymbol{\Phi}^{1/2}\boldsymbol{\Omega}^*$. It can be verified that for the optimal $\mathbf{F}$ the left unitary matrix $\mathbf{V}_F = \mathbf{V}$. Then (24) can be reformulated as

$$\begin{array}{ll} \min_{\boldsymbol{\Phi}} & \sum_{i=1}^K \phi_i \\ \text{subject to} & \{\lambda_{H,i}^2\phi_i + 1\}_{i=1}^L \succ_\times \{1+\rho_i\}_{i=1}^L. \\ & \phi_i \geq 0, \quad i = 1,\ldots,K. \end{array} \tag{25}$$

Let $\psi_i = \phi_i + \frac{1}{\lambda_{H,i}^2}$, for $i = 1,\ldots,K$, $\beta_i = \frac{1+\rho_i}{\lambda_{H,i}^2}$ for $i = 1,\ldots,K-1$, and $\beta_K = \frac{1}{\lambda_{H,K}^2}\prod_{i=K}^L(1+\rho_i)$. Then (25) can be further simplified to

$$\begin{array}{ll} \min_{\psi_i} & \sum_{i=1}^K \psi_i \\ \text{subject to} & \psi_1 \geq \beta_1 \\ & \ldots\ldots \\ & \psi_1\psi_2\ldots\psi_{K-1} \geq \beta_1\beta_2\ldots\beta_{K-1} \\ & \psi_1\psi_2\ldots\psi_K = \beta_1\beta_2\ldots\beta_K \\ & \psi_i \geq \frac{1}{\lambda_{H,i}^2}, \quad i = 1,2,\ldots,K \end{array} \tag{26}$$

Although $\rho_1 \geq \rho_2 \geq \ldots \geq \rho_L$, we note that it is not necessarily the case for $\beta_i, i = 1, \ldots, K$. Hence the constraints in (26) can *not* be written as $\{\psi_i\}_{i=1}^K \succ_\times \{\beta_i\}_{i=1}^K$. It can be shown that (26) can be solved recursively as shown in the following pseudo matlab code.

function $[\psi(1:K)]$ =acdPow$(\beta(1:K), \lambda_H(1:K))$
$\mu_k = \left(\prod_{i=1}^k \beta_i\right)^{1/k}, \quad k = 1, 2, \ldots, K$;
$l =$ argmax$_{1 \leq k \leq K} \;\; \mu_k$;
if $\mu_l \geq \frac{1}{\lambda_{H,l}^2}$
    $\psi_1 = \ldots = \psi_l = \mu_l$;
    if $l = K$
        return;
    end
    $[\psi(l+1:K)]$ =acdPow$(\beta(l+1:K), \lambda_H(l+1:K))$;
else
    $\psi_i = \frac{1}{\lambda_{H,i}^2}, \quad i = l, \ldots, K$;
    $\beta_{l-1} = \frac{\prod_{i=l-1}^K \beta_i}{\prod_{i=l}^K \psi_i}$;
    $[\psi(1:l-1)]$ =acdPow$(\beta(1:l-1), \lambda_H(1:l-1))$;
end

After determining the power loading level $\phi_i = \psi_i - \frac{1}{\lambda_{H,i}^2}$, we calculate $\mathbf{\Omega}$ and the nulling vectors $\{\mathbf{w}_i\}_{i=1}^L$ as we have done in Section 3. We note that for the case of $\psi_1 = \ldots = \psi_K = \mu_K$, where no self-invoking procedure happens, $\phi_i = \mu_K - \frac{1}{\lambda_{H,i}^2}, i = 1, \ldots, K$, is the standard "water filling" algorithm which maximizes the overall channel throughput. Otherwise, it turns out to be a multi-level "water filling" scheme which suffers from capacity loss.

5. NUMERICAL EXAMPLE

We present one numerical example to illustrate the effectiveness of the ACD scheme. We assume Rayleigh independent flat fading channels with $M_t = 5$ and $M_r = 6$. We consider equal QoS requirements for 5 independent substreams. Figure 1 compares the input power needed by our ACD scheme and the linear transceiver scheme of [3]. Our scheme has a uniform gain of about 2.5 dB for any SNR.

6. CONCLUSIONS

Based on the recently developed GTD matrix decomposition algorithm, we propose an ACD scheme utilizing the CSIT and CSIR, which can decompose a MIMO channel into multiple subchannels with prescribed capacities. We have also determined the subchannel capacity region such that a capacity lossless decomposition is possible. The applications of the ACD scheme for MIMO communications with QoS constraints are discussed. The numerical example verifies the advantage of ACD over its linear counterpart.

Finally, we remark that our further study shows that ACD can also be applied to optimal CDMA sequence designs [6].

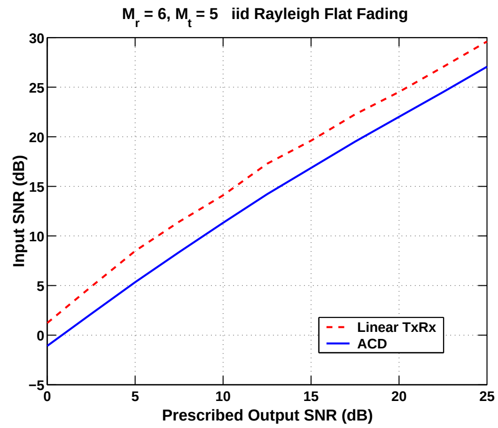

Fig. 1. Input SNR vs. Output SNR. Result is based on the average of 500 Monte Carlo trials of a i.i.d. Rayleigh flat fading channel with $M_t = 5$ and $M_r = 6$.

7. REFERENCES

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