The RF-Chain Limited MIMO System: Part II Case Study of V-BLAST and GMD

Yi Jiang   Mahesh K. Varanasi

Abstract

In Part I of this paper, we have established the fundamental D-M tradeoff of a RF-chain limited MIMO system as a function of transmitting antenna number M_t, receiving antenna number M_r, and $N \triangleq \min(L_t, L_r)$ where L_t and L_r are the numbers of the RF-chains at the transmitter and receiver respectively. Here we continue the investigation by studying two interesting schemes, i.e., the vertical bell-labs layered space time (V-BLAST) and the geometric mean decomposition (GMD) transceiver design, both applied to a RF-chain limited MIMO system with optimal antenna selection (AS). The V-BLAST scheme is popular in that it can achieve high spectral efficiency with simple scalar coding/decoding. However, it is also known to have maximal diversity gain only $M_r - M_t + 1$ when applied to a full $M_r \times M_t$ system. In this paper, we show that with optimal antenna selection, the diversity gain of V-BLAST can be greatly improved i.e., it can achieve the D-M tradeoff $d_{\mathrm{vb,p}}(r) = (M_r - N + 1)(M_t - N + 1)(1 - \frac{r}{N})$. Due to the analogy between the V-BLAST scheme in a MIMO channel and the successive interference cancellation (SIC) detector in a multi-access channel (MAC), our result sheds lights on the benefits of opportunistic communications facilitated by finite rate feedback. The GMD scheme has the same receiver as V-BLAST. However, it exploits the channel state information at transmitter (CSIT) to perform some channel-dependent unitary precoding. We derive the D-M tradeoff of the GMD scheme applied to the RF-Chain limited system. The GMD scheme is shown to be significantly better than the V-BLAST, although it is still D-M tradeoff suboptimal in general.

Keywords

MIMO, antenna selection, diversity gain, spatial multiplexing gain, tradeoff, outage probability, V-BLAST, geometric mean decomposition.

This work is supported in part by NSF Grants CCF-0423842 and CCF-0434410.
Y. Jiang is with Qualcomm, Inc. San Diego, CA 92126 (e-mail: yjiang.ee@gmail.com).
M. K. Varanasi is with the Department of Electrical and Computer Engineering, University of Colorado, Boulder, CO 80309-0425 USA (e-mail:varanasi@colorado.edu.) The material of this paper was presented in Asilomar Conference on Signals, Systems, and Computers 2006, IEEE International Conference on Communications 2007, and IEEE International Symposium on Information Theory 2007.

I. Introduction

This paper continues the D-M tradeoff study of RF-limited MIMO system by studying two interesting schemes, i.e., the vertical bell-labs layered space time (V-BLAST) [1] and the geometric mean decomposition (GMD) transceiver design [2] applied to a RF-chain limited MIMO system with antenna selection (AS).

It is well-known that V-BLAST can achieve the high spectral efficiency of a multi-input multi-output (MIMO) channel in that it can convert via successive interference cancellation (SIC) the MIMO channel into multiple parallel layers, through which the independently coded data substreams can be spatially multiplexed and be transmitted over the same time and frequency slot. However, the convenience of independent coding/decoding comes with poor D-M tradeoff performance. It was shown in [3] that in an $M_r \times M_t$ iid Rayleigh fading channel, the V-BLAST with independent coding/decoding on each layer can only achieve maximal diversity gain $M_r - M_t + 1$ even with optimal detection ordering. In this paper, we will show that the D-M tradeoff of V-BLAST gets much better when combined with channel-dependent AS. In particular, we prove that the V-BLAST combined with an optimal AS can achieve the D-M tradeoff $d_{\mathrm{vb,p}}(r) = (M_r - N + 1)(M_t - N + 1)\left(1 - \frac{r}{N}\right)$. where N is the smaller number of the RF-chains at the transmitter and receiver. Moreover, the V-BLAST combined with the fast AS algorithm introduced in Part I of this paper can achieve this diversity gain. Due to the analogy between the V-BLAST scheme in a MIMO channel and the successive interference cancellation (SIC) detector in a multi-access channel (MAC), our result gives insights into the multiuser diversity and the benefits of opportunistic user selection facilitated by finite rate feedback.

The GMD scheme shares the same detector as the V-BLAST, therefore it also admits independent coding/decoding. [1] By exploiting the channel state information at transmitter (CSIT), the GMD can achieve diversity gain $(M - K + 1)K$ [4, Section IV.C] where $M \triangleq \max(M_r, M_t)$ and $K \triangleq \min(M_r, M_t)$. When combined with the an optimal AS, the GMD scheme can achieve D-M tradeoff

$$d_{\mathrm{gmd,p}}(r) = \frac{(M_r - P)(M_t - P)}{N - P}(N - r) \tag{1}$$

where $P = \arg \min_{0 \le p \le N-1,\, p \in \mathbb{Z}} \dfrac{(M_r - p)(M_t - p)}{N - p}$. Clearly the GMD scheme is still suboptimal compared to the fundamental D-M tradeoff derived in Part I of this paper.

The rest of the paper is organized as following. In Section II, we present the D-M tradeoff of V-BLAST with antenna selection, the implication of the results to opportunistic multiuser communication is also discussed. In Section III, we derived the D-M tradeoff of the GMD scheme. The simulation results are presented in Section IV to validate our theoretical analysis. Section V concludes this paper.

[1] The GMD scheme also has a implementation form based on the dirty paper precoding[2].

II. Study of V-BLAST

In this section, we analyze the D-M gain tradeoff of the V-BLAST scheme [1] applied to RF-limited MIMO systems. The twin motivation for considering the V-BLAST architecture is because of its low complexity in MIMO links and also its applicability to multiple-access communication [5][6]. In the latter case, transmit antenna selection is equivalent to (channel-dependent or opportunistic) user selection and is made possible through a few bits of common feedback from the receiver to the transmitting users. A comparison with optimum decoder performance with receiver-only CSI reveals the benefit of such feedback both in terms of performance and complexity.

A. V-BLAST

The V-BLAST architecture is a simple and popular scheme capable of reaping a large portion of the high spectral efficiency of MIMO systems. When applied to the pruned channel, the V-BLAST architecture applies independent coding for L_t substreams with equal rate. The substreams are then transmitted simultaneously through the L_t selected transmit antennas. At the receiver side, the V-BLAST detects and decodes the substreams one by one through the *successive interference cancellation* (SIC) procedure. To make the SIC procedure work properly, the V-BLAST architecture should have more receive antennas than transmit antennas, i.e., $L_r \geq L_t$ $(= N)$.

Denote $\mathbf{H}_{\mathcal{S}_r,\mathcal{S}_t} \in \mathbb{C}^{L_r \times L_t}$ as the pruned channel matrix. Let $\mathbf{H}_{\mathcal{S}_r,\mathcal{S}_t} = \breve{\mathbf{Q}}\breve{\mathbf{R}}$ be the QR decomposition. If the zero forcing (ZF) interference cancellation is used, then after applying SIC, the ith data substream experiences a fading channel whose channel gain is $\breve{r}_{ii}$ which is the ith diagonal element of $\breve{\mathbf{R}}$. To study the D-M gain tradeoff V-BLAST with AS, we need to analyze the distributions of $\breve{r}_{ii}^2$ around origin, for which we have the following theorem.

Theorem II.1: Consider the iid Rayleigh channel given in [7, eq.(1)]. For any pruned channel matrix $\mathbf{H}_{\mathcal{S}_r,\mathcal{S}_t} \in \mathbb{C}^{L_r \times L_t}$, the following inequality holds

$$-\lim_{\epsilon \to 0_+} \frac{\log \mathbb{P}(\breve{r}_{ii}^2 < \epsilon)}{\log \epsilon} \leq (M_t - i + 1)(M_r - i + 1), \quad 1 \leq i \leq N. \tag{2}$$

Moreover, if the pruned channel is obtained through the proposed fast AS algorithm, then

$$-\lim_{\epsilon \to 0_+} \frac{\log \mathbb{P}(\breve{r}_{ii}^2 < \epsilon)}{\log \epsilon} = (M_t - i + 1)(M_r - i + 1), \quad 1 \leq i \leq N. \tag{3}$$

Proof: See Appendix. ■

Denote the overall multiplexing gain as r $(r \leq L_t)$. Each substream has multiplexing gain $\frac{r}{N}$. The ith substream is in outage if $\breve{r}_{ii}^2 < \rho^{\frac{r}{L_t}-1}$, which, according to Theorem II.1, has probability

$$P_{\text{outage,vb}}^{(i)} \doteq \rho^{(M_r-i+1)(M_t-i+1)(\frac{r}{L_t}-1)}$$

if the fast AS algorithm is used. Hence the overall outage probability of the V-BLAST is dominated by that of the Nth substream, i.e.,

$$P_{\text{outage,vb}} \doteq \rho^{(M_r-N+1)(M_t-N+1)(\frac{r}{N}-1)}.$$

Therefore the D-M gain tradeoff of the V-BLAST combined with the fast AS algorithm is

$$d_{\text{vb,p}}(r) = (M_r - N + 1)(M_t - N + 1)\left(1 - \frac{r}{N}\right). \tag{4}$$

Moreover, as implied by Theorem V.1 in Appendix, this tradeoff is also the upper bound to the V-BLAST with any AS approach. As a special case, when $N = M_t$,

$$d_{\text{vb,full}}(r) = (M_r - M_t + 1)\left(1 - \frac{r}{M_t}\right), \tag{5}$$

which implies that the maximal diversity gain of V-BLAST is $M_r - M_t + 1$. Indeed, it has been proven in [3] that the V-BLAST has maximal diversity gain $M_r - M_t + 1$ even with optimal detection ordering.

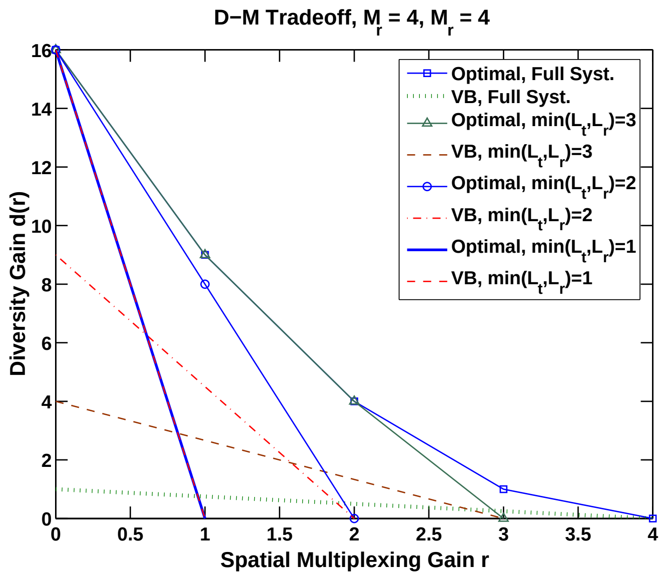

Fig. 1. D-M tradeoffs of the full (RF-chain non-limited) and pruned (RF-chain limited) V-BLAST systems

Figure 1 compares the D-M gain tradeoff of the *pruned* V-BLAST along with the optimal tradeoff. We note that antenna selection can improve the D-M tradeoff of V-BLAST at low multiplexing gain regime. The D-M tradeoff of V-BLAST is significantly worse than the optimum except for the special case $N = 1$.

In the recent work [8] where the same problem is considered except that the authors restrict their discussion to transmit AS only. Using a geometrical approach, the upper and lower bounds are given with respect to the maximal diversity gain of V-BLAST with transmit AS [8, Theorem II]:

$$(M_t - L_t + 1)(M_r - L_t + 1) \leq d_{\text{vb,p}} \leq (M_t - L_t + 1)(M_r - 1) \tag{6}$$

According to Theorem II.1, we see that the upper bound is in fact unattainable except for $L_t = 2$.

Note that the V-BLAST effectively decomposes a MIMO channel into multiple scalar subchannels/layers. It is known that the uncoded quadrature amplitude modulation (QAM) is universal over scalar channels [9]. Hence we conclude that the tradeoff given in (4) is achievable even using uncoded QAM, with frame length $T = 1$.

A.1 Implication for the Multiple-Access Channel and Opportunistic User Selection

Since V-BLAST is also applicable to the multi-access channel (MAC) [5][6] where multi-users communicate with the multi-antenna base station (BS), Theorem II.1 also has implications for the MAC. Consider a multi-access channel with M_t single-antenna users and the BS which has M_r antennas. In practice, M_t is usually far greater than M_r. Hence, to make V-BLAST work, only $L_t \leq M_r < M_t$ users are selected for simultaneous transmission. Such user selection is made possible by $\log_2 \binom{M_t}{L_t}$ bits of common feedback to all users. Thus transmit AS in MIMO links corresponds to user selection in the MAC. Through this opportunistic user selection and with each selected user assigned a common multiplexing gain of r/L_t, the MAC even with the *suboptimal* V-BLAST receiver has the D-M tradeoff $d_{\text{mac,opp}}(r) = (M_r - L_t + 1)(M_t - L_t + 1)(1 - \frac{r}{L_t})$.

It is interesting to compare the above scheme with what achieved in a MAC with just receiver CSI as obtained in [5][6]. Consider the case where L_t out of M_t users are selected in channel-independent round-robin fashion and suppose that a joint optimal decoder is used at the receiver. When $L_t \leq M_r$, the D-M gain tradeoff is simply $d_{\text{mac}}(r) = M_r(1 - \frac{r}{L_t})$. Hence, whenever the number of users M_t is greater than $M_r(M_r - L_t + 1)^{-1} + L_t - 1$, we have $d_{\text{mac,opp}}(r) > d_{\text{mac}}(r)$ which means that the D-M tradeoff performance of the system with opportunistic user selection but a sub-optimal successive cancellation based decoder uniformly dominates that of even the optimal decoder but with channel-independent user selection (with CSI only at receiver). The more users there are in the system, the greater the performance improvement and hence we have here a realization of *multiuser diversity*. Our result indicates that a huge performance gain *and* reduced complexity decoding can result due to the *finite rate feedback* (of $\log_2 \binom{M_t}{L_t}$ bits) which facilitate collaboration between the multiple users and the BS for opportunistic data transmission.

III. D-M Tradeoff of Geometric Mean Decomposition Scheme

In this section, we analyze the D-M tradeoff of the GMD scheme applied to a RF-chain limited system with AS. The GMD architecture is suitable for MIMO links with perfect CSI at receiver and transmitter [2].

A. Brief Introduction to GMD

The GMD scheme is based on the following theorem [10].

Theorem III.1: For any rank K matrix $\mathbf{H} \in \mathbb{C}^{M_r \times M_t}$ with singular values $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_K > 0$, there exists an upper triangular matrix $\mathbf{R} \in \mathbb{R}^{K \times K}$ with identical diagonal elements

$$r_{ii} = \bar{\lambda} \triangleq \left(\prod_{k=1}^{K} \lambda_k \right)^{\frac{1}{K}}, 1 \leq i \leq K, \tag{7}$$

and orthonormal matrices $\mathbf{Q} \in \mathbb{C}^{M_r \times K}$ and $\mathbf{P} \in \mathbb{C}^{M_t \times K}$, such that $\mathbf{H} = \mathbf{QRP}^*$.

A computationally efficient algorithm for the GMD matrix decomposition is given in [10]. Precoding the information vector $\mathbf{s} \in \mathbb{C}^K$ to be $\mathbf{x} = \mathbf{Ps}$ and applying the linear filter to the received data vector $\mathbf{y}$, we have

$$\tilde{\mathbf{y}} = \mathbf{Q}^* \mathbf{y} = \mathbf{Q}^* \mathbf{HPs} + \mathbf{Q}^* \mathbf{z} = \mathbf{Rs} + \tilde{\mathbf{z}}. \tag{8}$$

Here $\mathbf{R}$ is a upper triangular matrix with all the diagonal elements equal to the geometric mean of $\{\lambda_k\}_{k=1}^K$. Using decision feedback equalizer (DFE) at receiver or dirty paper precoder (DPP) at transmitter, we can remove the inter-substream interference due to the off diagonal elements of $\mathbf{R}$ and obtain K *identical* effective parallel subchannels with output SNR

$$\rho_{\text{gmd}} = \frac{\left(\prod_{k=1}^{K} \lambda_k \right)^{\frac{2}{K}} \rho}{K}. \tag{9}$$

Note that the GMD transceiver design requires feeding either full CSIT or the precoder matrix $\mathbf{P}$ from receiver to transmitter. We see that the AS helps reduce the overhead of feedback as it prunes the dimensionality of the channel.

B. D-M Tradeoff of GMD with AS

According to Theorem [7, Theorem III.1],

$$\frac{(M_t - N)!(M_r - N)!}{M_r! M_t!} \prod_{n=1}^{N} \lambda_n^2 \leq \prod_{n=1}^{N} \breve{\lambda}_n^2 \leq \prod_{n=1}^{N} \lambda_n^2 \tag{10}$$

Combined with AS, GMD converts the pruned channel into N subchannel with equal output SNR:

$$\rho_{\text{gmd}} = \left(\prod_{n=1}^{N} \breve{\lambda}_n^2 \right)^{1/N} \frac{\rho}{N}. \tag{11}$$

The outage probability of GMD scheme is

$$\begin{aligned} P_{\text{gmd,outage}} &= \mathbb{P}\left(\log(1 + \rho_{\text{gmd}}) < \frac{r}{N} \log(1 + \rho) \right) \\ &\doteq \mathbb{P}\left(\rho_{\text{gmd}} < \rho^{\frac{r}{N}} \right) \\ &\doteq \mathbb{P}\left(\prod_{n=1}^{N} \breve{\lambda}_n^2 < \rho^{r-N} \right) && (12) \\ \text{(by (10))} &\doteq \mathbb{P}\left(\prod_{n=1}^{N} \lambda_n^2 < \rho^{r-N} \right). && (13) \end{aligned}$$

Defining $\lambda_n = \rho^{-\alpha_n}$ and following the similar derivations leading to [7, eq. (43)], we obtain the D-M tradeoff of GMD as

$$\begin{aligned} d_{\mathrm{gmd,p}}(r) &= \inf_{\alpha_n} \sum_{n=1}^{N} c_n \alpha_n \\ \text{subject to} \quad & 0 < \alpha_1 \le \alpha_2 \le \cdots \le \alpha_N \\ & \sum_{n=1}^{N} \alpha_n > N - r, \end{aligned} \tag{14}$$

where

$$c_n = \begin{cases} M_t + M_r - 2n + 1 & n = 1, \cdots, N-1 \\ (M_t - N + 1)(M_r - N + 1) & n = N, \end{cases} \tag{15}$$

Comparing [7, eq. (43)] with (14) we see that the only difference between the two optimization problems is that α_N is *not* upper bounded in (14). The problem of (14) is also a convex optimization problem whose Lagrangian is

$$\mathcal{L}(\boldsymbol{\alpha}, \mu, \boldsymbol{\gamma}) = \sum_{n=1}^{N} c_n \alpha_n - \mu[\sum_{n=1}^{N} \alpha_n - (N - r)] - \gamma_1 \alpha_1 - \sum_{n=2}^{N} \gamma_n (\alpha_n - \alpha_{n-1}), \tag{16}$$

where the multipliers $\mu, \boldsymbol{\gamma} \ge 0$. Again, according to the convex optimization theory

$$d_{\mathrm{gmd,p}}(r) = \inf_{\boldsymbol{\alpha}} \sup_{\mu, \boldsymbol{\gamma}} \mathcal{L}(\boldsymbol{\alpha}, \mu, \boldsymbol{\gamma}). \tag{17}$$

According to the complementary slackness condition,

$$\gamma_1 \alpha_1 = \sum_{n=2}^{N} \gamma_n (\alpha_n - \alpha_{n-1}). \tag{18}$$

Equating to zero the partial derivative of the Lagrangian with respect to α_n we obtain the relations

$$c_n = \begin{cases} \mu + \gamma_n - \gamma_{n+1} & 1 \le n \le N-1 \\ \mu + \gamma_N & n = N. \end{cases} \tag{19}$$

Inserting (18) and (19) into (17) yields

$$\begin{aligned} d_{\mathrm{gmd,p}}(r) &= \inf_{\boldsymbol{\alpha}} \sup_{\mu, \boldsymbol{\gamma}} \textstyle\sum_{n=1}^{N-1} (\mu + \gamma_n - \gamma_{n+1}) \alpha_n + (\mu + \gamma_N) \alpha_N \\ &= \inf_{\boldsymbol{\alpha}} \sup_{\mu, \boldsymbol{\gamma}} \mu \textstyle\sum_{n=1}^{N} \alpha_n + \sum_{n=1}^{N-1} (\gamma_n - \gamma_{n+1}) \alpha_n + \gamma_N \alpha_N. \end{aligned} \tag{20}$$

Using the relations in (18) and some straightforward algebra, we obtain

$$\sum_{n=1}^{N-1} (\gamma_n - \gamma_{n+1}) \alpha_n + \gamma_N \alpha_N = 0. \tag{21}$$

Combining (20) and (21) we obtain

$$\begin{aligned} d_{\mathrm{gmd,p}}(r) &= \inf_{\boldsymbol{\alpha}} \sup_{\mu} \mu \textstyle\sum_{n=1}^{N} \alpha_n \\ &= \sup_{\mu} \mu (N - r). \end{aligned} \tag{22}$$

It follows from the N equations in (19) that μ is subject to the following N constraints:

$$N\mu + \gamma_1 = \sum_{n=1}^{N} c_n \tag{23}$$

and

$$\gamma_1 + n\mu - \sum_{i=1}^{n} c_i = \gamma_{n+1} \geq 0, \quad 1 \leq n \leq N-1. \tag{24}$$

Hence as visualized in [7, Figure 4], the line $f(r) = \mu + \gamma_1 r$ must (i) pass the Nth point due to the constraint of (23) and (ii) be above all the other $N-1$ points because of the N constraints in (24). We conclude that $\sup \mu$ corresponds to the slope of the line passing the Nth and the Pth points where

$$\begin{aligned} P = \quad & \arg\min_p \quad \tfrac{(M_r - p)(M_t - p)}{N-p} \\ & \text{s.t.} \qquad 0 \leq p \leq N-1, \; p \in \mathbb{Z} \end{aligned} \tag{25}$$

as given in [7, eq. (34)]. Therefore we have obtained the D-M tradeoff of GMD

$$d_{\text{gmd,p}}(r) = \frac{(M_r - P)(M_t - P)}{N - P}(N - r). \tag{26}$$

We compare the D-M tradeoff of GMD with AS against the optimal D-M tradeoff of the RF-chain non-limited (full) system in Figure 2. The full system is an iid Rayleigh channel with 5 transmit and 6 receive antennas.

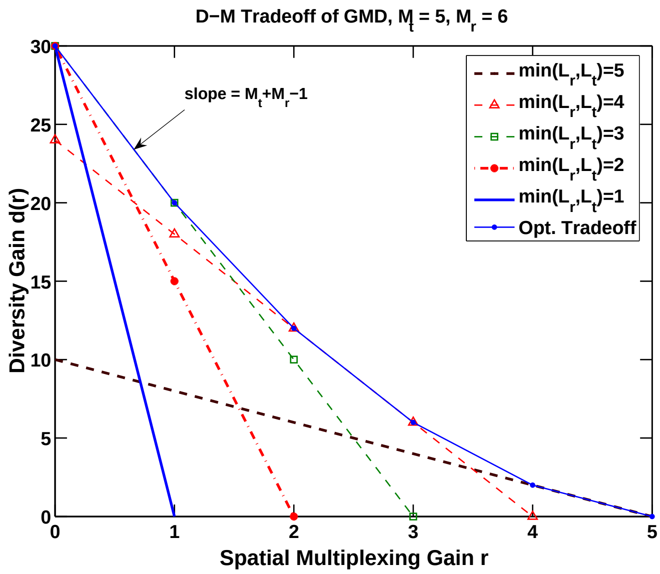

Fig. 2. The D-M tradeoff of the GMD scheme applied to a RF-chain limited system with AS vs. the optimal D-M tradeoffs of full MIMO system

Some observations are in order. (i) The D-M tradeoff of the GMD scheme is always a straight line connecting

$$\left(0, \frac{(M_r - P)(M_t - P)N}{N - P}\right), \quad \text{and} \quad (N, 0).$$

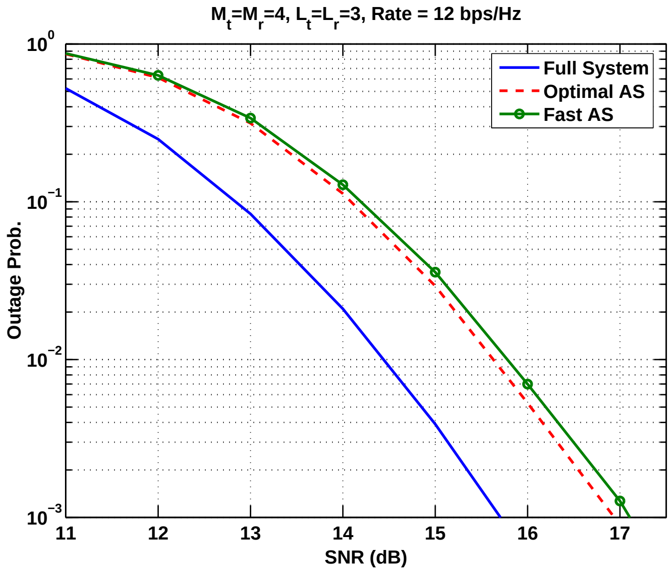

Fig. 3. Outage probabilities of the full MIMO system and the pruned system obtained via optimal AS and proposed fast AS algorithm

In particular, for a full system with $N = K$, then $P = K - 1$ and the GMD has the maximal diversity gain $(M - K + 1)K$, which agrees with the result in [4, Section IV.C]. (ii) The GMD has the same D-M tradeoff as the optimal one for multiplexing gain $r \le P$. (iii) Because the first piece of the optimal D-M tradeoff has slope $M_t + M_r - 1$, we see that if

$$\frac{M_t M_r}{N} \ge M_t + M_r - 1,$$

i.e.,

$$N \le \frac{M_t M_r}{M_t + M_r - 1},$$

then the D-M tradeoffs of GMD and the optimal are the same. (iv) The GMD is in general suboptimal because the scheme fails if the smallest singular value vanishes, which can be seen from (9). (v) Somewhat similar to the V-BLAST case, AS may improve the D-M tradeoff of the GMD scheme for low multiplexing gain, which can be explained by noting that AS makes the channel less ill-conditioned and hence improve the smallest singular value of the pruned channel matrix.

Similar to the argument preceding Section II-A.1, we can also conclude that the tradeoff of GMD given in (26) is achievable using uncoded QAM, with frame length $T = 1$.

IV. Numerical Examples

We present four numerical examples to validate the preceding theoretical analysis. We remark that the covariance of the input signal vector is constrained to be a scaled identity matrix for all the simulations.

The first example shows the near optimal performance of the proposed fast AS algorithm. Consider pruning a 4×4 iid Rayleigh channel into a 3×3 channel. Given the target rate 12 bps/Hz, Figure 3 compares the channel outage probabilities of the full system, the pruned system obtained using the proposed fast algorithm, and the optimally pruned one obtained through exhaustive search over $\begin{pmatrix} 4 \\ 3 \end{pmatrix} \times \begin{pmatrix} 4 \\ 3 \end{pmatrix} = 16$ combinations. We see that the performance of the proposed fast AS algorithm is very close to the optimal.

In the second example, we compare the outage probabilities $\mathbb{P}(r_{ii}^2 < \epsilon)$ and $\mathbb{P}(r_{ii,\max}^2 < \epsilon)$ in a 3-by-3 system. Here r_{ii} is the gain of the ith layer obtained via DFD using the greedy ordering rule, and $r_{ii,\max}$ is the maximum of r_{ii} over all the $M_t!$ permutations, for $i = 1, \ldots, M_t$. We run 10^5 Monte Carlo trials to obtain Figure 4. The probabilities $\mathbb{P}(r_{ii,\max}^2 < \epsilon)$, $i = 2, 3$ are the marked solid lines while $\mathbb{P}(r_{ii}^2 < \epsilon), i = 2, 3$ are represented by the marked dot lines. It is easy to see from [7, Section III] that the Greedy QR yields $r_{11} = r_{11,\max}$. Hence $\mathbb{P}(r_{11,\max}^2 < \epsilon) = \mathbb{P}(r_{11}^2 < \epsilon)$ and they are represented by the leftmost unmarked solid line. We may observe that the greedy ordering achieves the maximal diversity gains, which agrees with Theorem II.1. From the theoretical analysis, the diversity gains of the three layers are $\mathsf{D}_1 = 9, \mathsf{D}_2 = 4$ and $\mathsf{D}_3 = 1$. At first sight, one may see through comparing the two lines $-$ and $- \circ -$ that the diversity gain difference of r_{11} and $r_{22,\max}$ is seemingly smaller than the theoretical analysis: $\mathsf{D}_1 = 9, \mathsf{D}_2 = 4$. Indeed, with a large diversity gain, the outage probability curve approaches a vertical line and increasing the diversity gain further yields only marginal performance gain. It is important to note that there does *not* exist an ordering which can yield $r_{ii,\max}$ for each i simultaneously.

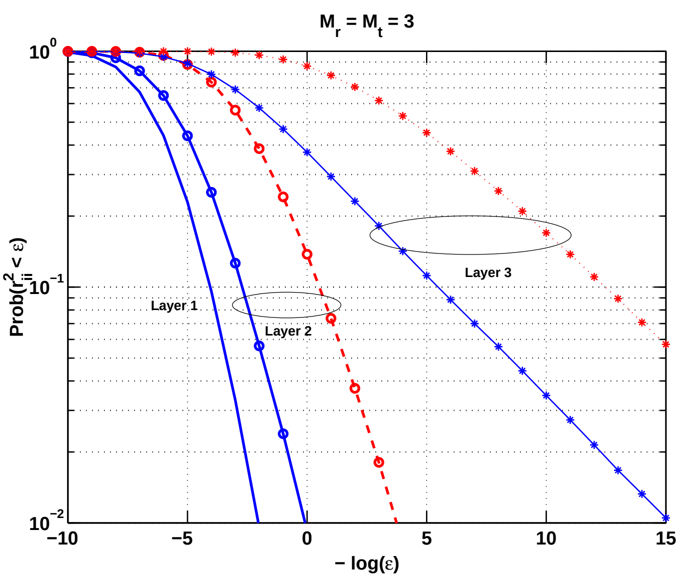

Fig. 4. The outage probabilities of the layers. The solid lines stand for the outage probability $\mathbb{P}(r_{ii,\max}^2 < \epsilon)$ and the marked dot lines represent $\mathbb{P}(r_{ii}^2 < \epsilon)$.

In the third example, we applies the greedy AS algorithm to a MAC channel as user selection (US). Consider a MAC channel where there are M_t ($M_t = 10$) single-antenna users and a base-station with five receiving antennas. We consider two schemes. For one L_t ($L_t = 5$) users are randomly selected and a ML receiver is deployed. For the other, L_t users are selected by the greedy AS algorithm and the receiver is an ordered V-BLAST scheme [11]. Figure 5 compares the two scheme. We see that when combined with the US algorithm, the simple and suboptimal V-BLAST scheme can even outperform the optimal ML receiver but with random US. Indeed, here the ML receiver can achieve the maximal diversity gain $M_r = 5$ [6] while the V-BLAST combined with greedy US can achieve the maximal diversity gain $d = (M_r - L_r + 1)(M_t - L_t + 1) = 6$.

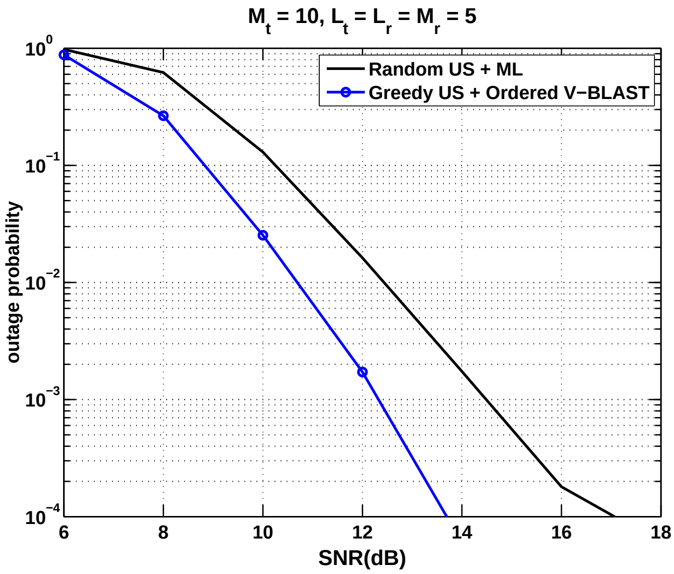

Fig. 5. The outage probabilities of MAC with ML receiver and random user selection (US) versus the ordered V-BLAST scheme cobmined with greedy user US. The sum rate is 10 bps/Hz and is equally distributed among five users.

In the last example, we compare the outage probabilities of GMD architecture and the optimal system (with CSI at transmitter and receiver) with and without AS. The channel is an i.i.d. Rayleigh fading channel with $M_t = 5$ and $M_r = 6$. The target rate is 14 bps/Hz. Figure 6 shows the significant performance gap between the optimal and the GMD schemes in the full system. In particular, the optimal system has much higher diversity gain than the GMD scheme. According to the D-M tradeoff analysis, GMD has a maximal diversity gain of $(M_r - M_t + 1)M_t = 10$ while the optimal system has a maximal diversity gain of $M_t M_r = 30$. By applying the fast AS algorithm, we obtain a pruned channel with $L_t = L_r = 3$. In such a system, the GMD and the optimal systems have almost the same outage probabilities. This result agrees with the D-M gain tradeoff analysis. Indeed, it can be seen from [7, Fig. 3] and 2 that the GMD and optimal systems have the same D-M tradeoff when $N = 3$.

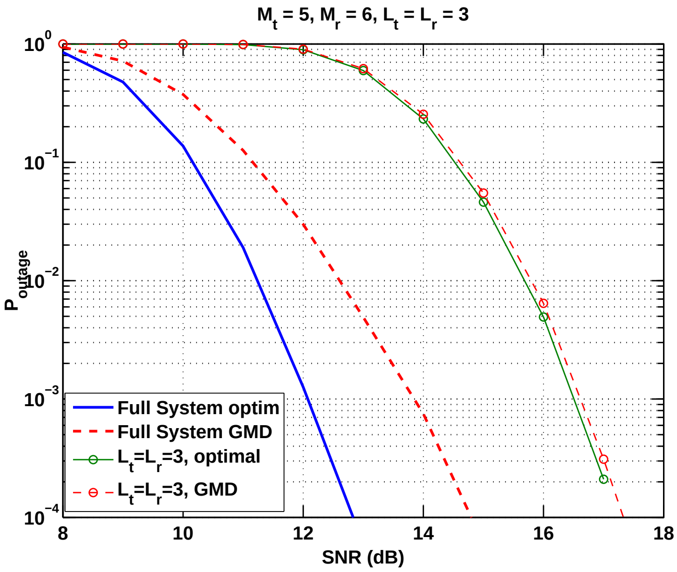

Fig. 6. The outage probability of GMD and optimal system with and without AS. $M_t = 5$, $M_r = 6$, $L_r = L_t = 3$, and target rate $R = 14$ bps/Hz.

V. Conclusion and Discussions

In this two-part paper we have studied the fundamental diversity-multiplexing (D-M) tradeoff of a MIMO system with antenna selection (AS) at both transmitter and receiver side. We show that a MIMO system with AS has the same D-M tradeoff as the full system if the multiplexing gain is less than some threshold P, although it can suffer from considerable diversity gain loss for the multiplexing gain $r \geq P$. Our analysis yields a fast yet D-M tradeoff-optimal AS algorithm. We also study the D-M tradeoffs of the V-BLAST and GMD space-time architectures applied to pruned channels. We show that for both V-BLAST and GMD, the pruned channel may have better D-M tradeoff than the full system in the low multiplexing gain regime.

The D-M tradeoff analysis may provide useful guidance for practical system design in two aspects. First, it enables us to quantify the D-M tradeoff loss relative to the full system and hence quantify the price paid for reduced hardware and computational complexity. The same analysis can also be viewed as the D-M tradeoff *improvement* due to introducing additional transmit and receiver antennas but with no increase of the hardware/software complexity through antenna selection. Second, just transmit antenna selection within the V-BLAST architecture corresponds to the problem of user selection in a multiple-access channel with a nulling-successive cancellation decoder. Our results here indicate that a D-M gain tradeoff that is superior to that achievable with the optimum decoder can be obtained in spite of the sub-optimality of the V-BLAST decoder, thanks to a few bits of feedback broadcast to all users as to which of them is to transmit based on the realized channels of all users at the common receiver. Combined with such results, we can determine the number of the selected antennas. If the system operates at the multiplexing gain r, we can determine the smallest number N such that $P \geq r$. For instance, as we shown in [7, Fig. 3], for a 6×5 channel if $2 < r < 3$, then $L_t = L_r = 4$ is sufficient.

Appendix

We have proven in [12] the following theorem.

Theorem V.1: Consider the ordered QR decomposition $\mathbf{H\Pi} = \mathbf{QR}$ where $\mathbf{\Pi}$ is a permutation matrix *dependent* on $\mathbf{H}$. Let r_{ii} be the ith diagonal of $\mathbf{R}$. The inequality

$$-\lim_{\epsilon \to 0_+} \frac{\log \mathbb{P}(r_{ii}^2 < \epsilon)}{\log \epsilon} \leq (M_t - i + 1)(M_r - i + 1), \quad 1 \leq i \leq N, \tag{27}$$

holds for any channel-dependent permutation matrix $\mathbf{\Pi}$.

Denote $\mathbf{H}_{:,\mathcal{S}_t} \in \mathbb{C}^{M_r \times L_t}$ as the channel matrix after transmit AS, and $\mathbf{H}_{:,\mathcal{S}_t} = \tilde{\mathbf{Q}}\tilde{\mathbf{R}}$ as its QR decomposition. Then according to Theorem V.1, for any $\mathcal{S}_t$ the diagonal elements of $\tilde{\mathbf{R}}$ satisfy

$$-\lim_{\epsilon \to 0_+} \frac{\log \mathbb{P}(\tilde{r}_{ii}^2 < \epsilon)}{\log \epsilon} \leq (M_t - i + 1)(M_r - i + 1), \quad 1 \leq i \leq L_t (= N). \tag{28}$$

For any submatrix of $\mathbf{H}_{:,\mathcal{S}_t}$, which we denote as $\mathbf{H}_{\mathcal{S}_t,\mathcal{S}_t}$ whose QR decomposition is $\mathbf{H}_{\mathcal{S}_r,\mathcal{S}_t} = \breve{\mathbf{Q}}\breve{\mathbf{R}}$, it is routine to show that $\breve{r}_{ii}^2 \leq \tilde{r}_{ii}^2$ for $1 \leq i \leq N$. Therefore, it follows from (28) that

$$-\lim_{\epsilon \to 0_+} \frac{\log \mathbb{P}(\breve{r}_{ii}^2 < \epsilon)}{\log \epsilon} \leq (M_t - i + 1)(M_r - i + 1), \quad 1 \leq i \leq N. \tag{29}$$

Now the first part of Theorem II.1 is proven.

To prove the second part of Theorem II.1, we first recall the theorem implied in [13].

Lemma V.2 (See [13]) Consider the iid Rayleigh fading channel $\mathbf{H}$ given in [7, eq.(1)] with ordered singular values $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{M_t} > 0$. Then

$$-\lim_{\epsilon \to 0_+} \frac{\log \mathbb{P}(\lambda_i^2 < \epsilon)}{\log \epsilon} = (M_t - i + 1)(M_r - i + 1), \quad 1 \leq i \leq N. \tag{30}$$

According to Theorem [7, Theorem III.1], $\nu_i \lambda_i \leq \breve{\lambda}_i \leq \lambda_i$ for some positive constant ν_i. Hence

$$-\lim_{\epsilon \to 0_+} \frac{\log \mathbb{P}(\breve{\lambda}_i^2 < \epsilon)}{\log \epsilon} = -\lim_{\epsilon \to 0_+} \frac{\log \mathbb{P}(\lambda_i^2 < \epsilon)}{\log \epsilon} = (M_t - i + 1)(M_r - i + 1), \quad 1 \leq i \leq N. \tag{31}$$

Denote $\mathbf{H}_{\mathcal{S}_r,\mathcal{S}_t}\breve{\mathbf{\Pi}} = \breve{\mathbf{Q}}\breve{\mathbf{R}}$ the greedy QR decomposition. It follows from Lemma [7, Lemma III.3] that $\breve{r}_{ii}^2 \geq \frac{\breve{\lambda}_i^2}{L_t - i + 1}$. Hence

$$-\lim_{\epsilon \to 0_+} \frac{\log \mathbb{P}(\breve{r}_{ii}^2 < \epsilon)}{\log \epsilon} \geq -\lim_{\epsilon \to 0_+} \frac{\log \mathbb{P}(\breve{\lambda}_i^2 < \epsilon)}{\log \epsilon} = (M_t - i + 1)(M_r - i + 1), \quad 1 \leq i \leq N. \tag{32}$$

Combining (32) and (29), we have proven the second part of Theorem II.1.

References

[1] G. J. Foschini, G. D. Golden, R. A. Valenzuela, and P. W. Wolniansky, "Simplified processing for high spectral efficiency wireless communication employing multiple-element arrays," *Wireless Personal Communications*, vol. 6, pp. 311–335, March 1999.

[2] Y. Jiang, J. Li, and W. Hager, "Joint transceiver design for MIMO communications using geometric mean decomposition," *IEEE Transactions on Signal Processing*, vol. 53, pp. 3791 – 3803, October 2005.

[3] Y. Jiang, M. Varanasi, and J. Li, "Performance analysis of ZF and MMSE equalizers for MIMO systems: A closer study in high SNR regime," *IEEE Transactions on Information Theory*, accepted. 2008.

[4] Y. Jiang, J. Li, and W. Hager, "Uniform channel decomposition for MIMO communications," *IEEE Transactions on Signal Processing*, vol. 53, pp. 4283 – 4294, November 2005.

[5] N. Prasad and M. K. Varanasi, "Outage analysis and optimization for multiaccess/V-BLAST architecture over MIMO Rayleigh fading channels," *41st Annual Allerton Conf. on Comm. Control, and Comput., Monticello, IL*, October 2003.

[6] D. Tse, P. Viswanath, and L. Zheng, "Diversity-multiplexing tradeoff in multiple access channel," *IEEE Transactions on Information Theory*, vol. 50, pp. 1859–1874, September 2004.

[7] Y. Jiang and M. Varanasi, "The RF-chain limited mimo system: Part I optimum diversity-multiplexing tradeoff," *IEEE Transactions on Wireless Communication*, accepted.

[8] H. Zhang, H. Dai, Q. Zhou, and B. L. Hughes, "On the diversity order of spatial multiplexing systems with transmit antenna selection: A geometrical approach," *IEEE Transactions on Information Theory*, vol. 52, pp. 5297–5311, December 2006.

[9] S. Tavildar and P. Viswanath, "Approximately universal codes over slow fading channels," *IEEE Transactions on Information Theory*, vol. 52, pp. 3233–3258, July 2006.

[10] Y. Jiang, W. Hager, and J. Li, "The geometric mean decomposition," *Linear Algebra and Its Applications*, vol. 396, pp. 373–384, February 2005.

[11] G. J. Foschini, D. Chizhik, M. J. Gans, C. Papadias, and R. A. Valenzuela, "Analysis and performance of some basic space time architectures," *IEEE Journal on Selected Areas in Communications*, vol. 21, pp. 303–320, April 2003.

[12] Y. Jiang and M. Varanasi, "Spatial multiplexing architectures with jointly designed rate-tailoring and ordered V-BLAST decoding Part I: Diversity-multiplexing trade-off ananlysis," *IEEE Transactions on Wireless Communications*, vol. 8, pp. 3252–3261, July 2008.

[13] L. Zheng and D. Tse, "Diversity and multiplexing: A fundamental tradeoff in multiple-antenna channels," *IEEE Transactions on Information Theory*, vol. 49, pp. 1073–1096, May 2003.