EEL 3135 Quiz 4

Name: ____________________________ UFID: ____________________________

1. Suppose an LTI system is described by the following difference equation:

\[ y[n] = x[n] + 2x[n - 2] + x[n - 4] \]

Find the system transfer function \( H(z) \).


\[ \implies H(z) = 1 + 2z^{-2} + z^{-4} + \frac{1}{z} \]

2. Now suppose a specific input \( x_1[n] \) is applied to the system. Find the resulting output \( y_1[n] \).

\[ x_1[n] = 3 + 2\delta[n - 1] + \cos[\pi n + 3\pi/4] \]

\[ H(e^{j\omega}) = 1 + 2e^{-2j\omega} + e^{-j\omega} = e^{-j\omega} \left[ 2 + 2\cos(2\omega) \right] \]

\[ H(e^{j0}) = e^{-j0} \left[ 2 + 2\cos(2\cdot0) \right] = 4 \]

\[ H(e^{j\pi}) = e^{-j\pi} \left[ 2 + 2\cos(2\cdot\pi) \right] = 4 + 2 \]

\[ y_1[n] = 3 \cdot H(e^{j0}) + 2h[n-1] + H(e^{j\pi}) \cdot \cos(\pi n + 3\pi/4) \]

\[ y_1[n] = 12 + 2\delta[n-1] + 4\delta[n-2] + 2\delta[n-3] + 4\cos(\pi n + 3\pi/4) \]

Bonus: Are there any frequencies that are blocked (i.e. \( H(e^{j\omega}) = 0 \) for this particular \( \omega \)) by the system?

\[ \text{Yes. } \quad \omega = \pi/2 \]

\[ H(e^{j\pi/2}) = e^{j\pi/2} \left[ 2 + 2\cos(2\cdot\pi/2) \right] = 0 \]
EEL 3135 Quiz 4

1. Suppose an LTI system is described by the following difference equation:

\[ y[n] = \frac{1}{2} x[n] + x[n - 2] + \frac{1}{2} x[n - 4] \]

Find the system transfer function \( H(z) \).

\[ h[n] = \frac{1}{2} \delta[n] + \delta[n - 2] + \frac{1}{2} \delta[n - 4] \]

\[ H(z) = \frac{1}{2} + z^{-2} + \frac{1}{2} z^{-4} \]

2. Now suppose a specific input \((x_1[n])\) is applied to the system. Find the resulting output \((y_1[n])\).

\[ x_1[n] = 1 + 3 \delta[n - 2] + 2 \cos \left( \pi n + \frac{\pi}{6} \right) \]

\[ H(e^{j\omega}) = \frac{1}{2} + e^{-2j\omega} + \frac{1}{2} e^{-j\omega} = e^{j\omega} \left[ \frac{1 + \cos(2\omega)}{2} \right] \]

\[ H(e^{j0}) = e^{0} \left[ 1 + \cos(0) \right] = 2 \]

\[ H(e^{j\pi}) = e^{-j\pi} \left[ 1 + \cos(2\pi) \right] = 2 \]

\[ y_1[n] = y_1 \cdot H(e^{j\omega}) + 3 h[n - 2] + H(e^{j\pi}) \cdot 2 \cos \left( \pi n + \frac{\pi}{6} \right) \]

\[ y_1[n] = 2 + 3 \delta[n - 2] + 3 \delta[n - 4] + \frac{3}{2} \delta[n - 6] + 4 \cos \left( \pi n + \frac{\pi}{6} \right) \]

Bonus: Are there any frequencies that are blocked (i.e. \( H(e^{j\omega}) = 0 \) for this particular \( \omega \)) by the system?

Yes. If \( \omega = \pi / 2 \)

\[ H(e^{j\pi/2}) = e^{-j\pi/2} \left[ 1 + \cos(2 \cdot \pi / 2) \right] = 0 \]
EEL 3135 Quiz 4

1. Suppose an LTI system is described by the following difference equation:

\[ y[n] = \frac{3}{2} x[n] + 3x[n - 2] + \frac{3}{2} x[n - 4] \]

Find the system transfer function \( H(z) \).

\[ h[n] = \frac{3}{2} \delta[n] + 3 \delta[n-2] + \frac{3}{2} \delta[n-4] \]

\[ H(z) = \frac{3}{2} + 3z^{-2} + \frac{3}{2} z^{-4} \]

2. Now suppose a specific input \( x_1[n] \) is applied to the system. Find the resulting output \( y_1[n] \).

\[ x_1[n] = 5 + 2 \delta[n - 3] + 6 \cos[\pi n + \frac{5\pi}{6}] \]

\[ H(e^{j\omega}) = \frac{3}{2} + 3 e^{-2j\omega} + \frac{3}{2} e^{-4j\omega} = e^{j\omega} \left[ 3 + 3 \cos(2\omega) \right] \]

\[ H(e^{j\omega}) = e^{-2j\omega} \left[ 3 + 3 \cos(2\omega) \right] = 6 \]

\[ H(e^{j\pi/2}) = e^{-2j\pi/2} \left[ 3 + 3 \cos(2\cdot\pi/2) \right] = 6 \]

\[ y_1[n] = H(e^{j\omega}) \cdot 5 + 2h[n-3] + H(e^{j\pi/2}) \cdot 6 \cos(\pi n + \frac{5\pi}{6}) \]

\[ y_1[n] = 30 + 3 \delta[n-3] + \frac{6}{9} \delta[n-5] + 3 \delta[n-7] + 36 \cos(\pi n + \frac{5\pi}{6}) \]

Bonus: Are there any frequencies that are blocked (i.e. \( H(e^{j\omega}) = 0 \) for this particular \( \omega \)) by the system?

Yes. If \( \omega = \pi/2 \)

\[ H(e^{j\pi/2}) = e^{-2j\pi/2} \left[ 3 + 3 \cos(2\cdot\pi/2) \right] = 0 \]