Homework 2

EEL6533

Due date: September 20, 2011

1. One of the two possible sources supplies the inputs to the simple communication channel as shown:

\[ H_1 : P(1) = 0.5, P(0) = 0.5; \]
\[ H_0 : P(1) = 0.6, P(0) = 0.4. \]

a) Find a decision rule that maximizes \( P_D \) subject to the constraint that \( P_F \leq 0.48 \). Determine \( P_D \) and \( P_F \).

b) Find a decision rule that maximizes \( P_D \) subject to \( P_F \leq 0.4 \). Determine \( P_D \) and \( P_F \).

2. Consider the following binary hypothesis test:

\[ H_1 : Y = K + N, \]
\[ H_0 : Y = N. \]

Plot the ROC curves for \( K = 0, 1, 2, 3 \).

3. Consider M-ary (M=3) Hypothesis Test. Assume that \( \Pi_0 = P(H_0), \Pi_1 = P(H_1), \) and \( \Pi_2 = P(H_2) \) (\( \Pi_0 + \Pi_1 + \Pi_2 = 1 \)) are known. Assume that the cost of say \( H_i \) when \( H_j \) is true is \( C_{ij} \).
a) Find the decision that minimizes the risk:

\[
\text{Risk} = \sum_{i=0}^{2} \sum_{j=0}^{2} \Pi_j C_{ij} \int_{\Gamma_i} f(y|H_j) dy.
\]

b) Let

\[
C_{ij} = \begin{cases} 
0 & \text{if } i = j, \\
1 & \text{if } i \neq j,
\end{cases}
\]

\[
\Pi_0 = \Pi_1 = \Pi_2 = \frac{1}{3}.
\]

\[
H_0 : Y = -1 + N,
\]

\[
H_1 : Y = 1 + N,
\]

\[
H_2 : Y = 2 + N,
\]

and

\[
N \sim \mathcal{N}(0,1).
\]

Find the best decision rule.