1. (a) \( r(-1) = r^*(1) = -10 \)
   \( r(-2) = r^*(2) = 9 \)
   \( r(-3) = r^*(3) = -8 \)

\[
\begin{bmatrix}
  r(0) & r(-1) & r(-2) \\
  r(1) & r(0) & r(-1) \\
  r(2) & r(1) & r(0)
\end{bmatrix}
\begin{bmatrix}
  1 \\
  0 \\
  0
\end{bmatrix}
= \begin{bmatrix}
  \sigma^2 \\
  0 \\
  0
\end{bmatrix}
\]

\[\Rightarrow \begin{cases}
  r(0) + (-10)\alpha = \sigma^2 \\
  (-10) + \alpha \cdot r(0) = 0 \\
  9 + (-10)\alpha = 0
\end{cases}\]

\[\Rightarrow \begin{cases}
  r(0) = \frac{100}{9} \\
  \alpha = \frac{9}{10} \\
  \sigma^2 = \frac{19}{9}
\end{cases}\]

(b) \( x(n) + \alpha x(n-1) = u(n) \)

The transfer function is

\[H(jw) = \frac{1}{1 + \alpha e^{-jw}}\]

\[P_x(w) = |H(jw)|^2 \sigma^2\]

\[= \frac{\sigma^2}{1 + 2\alpha \cos w + \alpha^2}\]

\[= \frac{\frac{19}{9}}{1 + \frac{18}{9} \cos w + 0.81}\]

2. (a) \( k = \theta_1 = \frac{-r(1)}{r(0)} \)

\[\Rightarrow r(1) = -k \cdot r(0) = -0.5 \]

\[\sigma_1^2 = r(0) - |r(1)|^2 / r(0)\]

\[= 1 - 0.5^2 / 1\]

\[= 0.75\]

\[\hat{\gamma}_1 = \frac{r(1)}{\sigma_1^2} = -0.5 \]

\[k_2 = -\frac{r(2) + \hat{\gamma}_1 \theta_1}{\sigma_1^2}\]

\[\Rightarrow r(2) = -k_2 \sigma_1^2 - \hat{\gamma}_1 \theta_1 = -0.5 \times 0.75 - (-0.5) \times 0.5 = -0.125\]

\[\text{for } i \geq 3\]
\[ \sigma^2 = \sigma^2_1 = \sigma^2_1 (1 - |h_1|^2) \\
= 0.75 (1 - 0.5^2) \\
= 0.5625 \]

(b) \[
\theta_2 = \begin{bmatrix} \theta_1 \\ 0 \end{bmatrix} + h_2 \begin{bmatrix} \tilde{\theta}_1 \\ 1 \end{bmatrix} \\
= \begin{bmatrix} 0.15 \\ 0 \end{bmatrix} + 0.5 \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} \\
= \begin{bmatrix} 0.75 \\ 0.5 \end{bmatrix} \\
\Rightarrow \sigma_1 = 0.75 \\
\sigma_2 = 0.5
\]

(c) The transfer function is
\[ H(z) = \frac{1}{1 + 0.75z^{-1} + 0.5z^{-2}} \]

The poles are
\[ z_1 = -0.75 + 1.2j, \quad z_2 = -0.75 - 1.2j \]
\[ |z_1| < 1, \quad |z_2| < 1 \]

\[ \Rightarrow \text{The poles of the AR(2) model are inside the unit circle.} \]

3. (a) \[ s_1(n) = y_1 e^{j2\pi f_1 n} \]
\[ s_2(n) = y_2 e^{j2\pi f_2 n} \]

\[ A(z) = (1 - e^{j2\pi f_1} z^{-1}) (1 - e^{j2\pi f_2} z^{-1}) \]

\[ \begin{bmatrix} x(1) \\ x(2) \\ x(3) \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \]

\[ \begin{bmatrix} -0.7588 + j0.1420 \\ -0.1420 + j0.8968 \\ -0.7588 + j0.1420 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0.1420 + j0.8968 \\ -1.7601 - j0.2788 \end{bmatrix} \]

\[ \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0.4999 - j1.5389 \\ -0.8090 - j0.5878 \end{bmatrix} \]
\[ A(z) = 1 + a_1 z^{-1} + a_2 z^{-2} \]

The roots of \( A(z) \) are
\[ z_1 = -0.8089 + j0.5877 \]
\[ z_2 = 0.3090 + j0.9512 \]
\[ f_1 = \text{ang}(z_1)/2\pi = 0.4 \]
\[ f_2 = \text{ang}(z_2)/2\pi = 0.2 \]

\[
\begin{bmatrix} 1 & 1 \\ \exp(j2\pi f_1) & \exp(j2\pi f_2) \end{bmatrix} \begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} x(0) \\ x(1) \end{bmatrix}
\]

(b) \[ A(z) = (1 - e^{-j2\pi f_1} z^{-1})(1 - e^{-j2\pi f_2} z^{-1}) = 1 - (e^{-j2\pi f_1} + e^{-j2\pi f_2}) z^{-1} + e^{-j2\pi f_1} e^{-j2\pi f_2} z^{-2} \]

\[ \begin{bmatrix} x(1) & x(0) \end{bmatrix} \begin{bmatrix} e^{j2\pi f_1} + e^{j2\pi f_2} \\ -e^{j2\pi f_1} e^{j2\pi f_2} \end{bmatrix} = x(2) \]

\[ x(1) (e^{j2\pi f_1} + e^{j2\pi f_2}) - e^{j2\pi f_1} e^{j2\pi f_2} x(0) = x(2) \]

\[ e^{j2\pi f_2} = \frac{x(2) - e^{j2\pi f_1} x(1)}{x(1) - e^{j2\pi f_1} x(0)} \]

\[ f_2 = \frac{1}{j2\pi} \ln \left( \frac{x(2) - e^{j2\pi f_1} x(1)}{x(1) - e^{j2\pi f_1} x(0)} \right) \]

\[ = \frac{1}{j2\pi} \ln \left( \frac{0.1420 + j0.8968 - e^{j2\pi 0.2} \cdot (0.2788 + j0.1420)}{0.2788 + j0.1420 - e^{j0.4\pi} (1 + j)} \right) \]

\[ = 0.4 \]